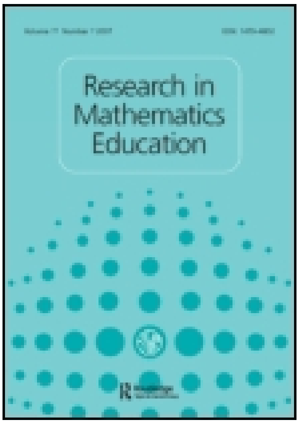

Research in Mathematics Education
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/rrme20

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Version of record first published: 20 Sep 2008.

To cite this article: Shelly Dole & Mal Shield (2008): The capacity of two Australian eighth-grade textbooks for promoting proportional reasoning, Research in Mathematics Education, 10:1, 19-35

To link to this article: http://dx.doi.org/10.1080/14794800801915863

Research in Mathematics Education
Vol. 10, No. 1, March 2008, 19–35

The capacity of two Australian eighth-grade textbooks for promoting proportional reasoning

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In the middle grades mathematics curriculum, proportional reasoning is fundamental to many topics, and hence, there is considerable scope for instruction to focus on assisting students to recognize the proportional structure of many problem situations. Yet, proportional reasoning, and particularly the study of ratio and proportion, pose considerable difficulty for students. Drawing on proportional reasoning literature as well as mathematics curriculum reform principles underpinning the current mathematics syllabus documents in Queensland, Australia, we devised criteria for textbook analysis, and applied these to eighth-grade Australian mathematics textbooks, to explore the degree to which knowledge connections and proportional reasoning were promoted. The analysis revealed a predominance of calculation procedures, with relatively few tasks and explanations to support conceptual understanding.

Keywords: textbooks; proportional reasoning; multiplicative structures

Introduction

Many mathematical tasks and activities require proportional reasoning. Drawing a map of a pathway from home to school; determining the better buy when 1 kg costs $3.50 and 1.5 kg costs $4.20; determining whether there is more chance of selecting red from a collection of three red and four blue compared to a collection of six red and eight blue: all require proportional reasoning. Proportional reasoning is required to operate fully within the domains of scale, probability, percent, rate, trigonometry, the geometry of plane shapes, algebra and fractions. The development of proportional reasoning is a gradual process, underpinned by increasingly more sophisticated multiplicative thinking and the ability to compare two quantities in relative (multiplicative) rather than absolute (additive) terms (Lamon 1994). According to Lesh, Post and Behr (1988), proportional reasoning is a key concept underlying a wide range of topics studied in the middle grades (ages 10 to 14 years). The task for middle grade teachers is to assist students to build, consolidate and link their proportional reasoning ability; not an easy task, as research consistently indicates students' difficulty with proportion related topics (Behr et al. 1992; Ben-Chaim et al. 1998; Lo and Watanabe 1997).

This paper reports on an exploration of textbook material relating to ratio and proportion. In this study, two popular textbooks for eighth-grade (approximately 12 years) students used in Australian schools were analysed using criteria drawn from

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ISSN 1479-4802 print/ISSN 1754-0178 online

DOI: 10.1080/14794800801915863
http://www.informaworld.com

proportional reasoning literature and mathematics curriculum reform principles underpinning the current syllabus documents. The analysis focused on the extent to which the books supported and assisted learners to connect and integrate mathematical ideas, specifically in relation to the topics of ratio and proportion.

The concept of proportion

Proportional reasoning and multiplicative thinking

Proportional reasoning is to understand the multiplicative relationship inherent in situations of comparison (Behr et al. 1992). Intuitive proportional reasoning is fostered through young children's early experiences with sharing and making groups, for example, pouring and counting the number of cups of water to fill a bucket at bath time. Being able to state that if three cups of water fills one bucket, then nine cups would fill three is an example of proportional reasoning based on multiplication. As stated by Cramer, Post and Currier (1993, 160), "the critical component of proportional situations is the multiplicative relationship that exists among the quantities that represent the situation."

Being able to distinguish between additive and multiplicative situations is the basis for being able to determine proportional from non-proportional situations. Research has shown that many students have difficulty making such a distinction (Cramer, Post and Currier 1993; Van Dooren et al. 2005). Hence, students require opportunities and rich learning experiences to promote their conceptual understanding of proportion and to be able to differentiate proportion from non-proportion situations in the real world. This capacity is developmental, becoming more stable with maturity and experience (Clark and Kamii 1996), but even students at eighth grade still experience difficulty in distinguishing proportional from non-proportional situations (Van Dooren et al. 2005). Students' intuitive strategies for solving proportion problems are typically additive (Hart 1981). Thus, for example, students regard a ratio of 4:7 as proportional to 2:5. Conversely, students will often blindly use a proportion equation to solve problems that are not proportional. Cramer, Post and Currier's (1993) oft-cited running track problem (Sue and Julie are running equally fast around a running track. Sue started first. After nine laps, Julie had run three. When Julie has run 15, how many has Sue run?) is evidence of this. Proportional reasoning entails the ability to distinguish additive from multiplicative comparisons, and knowing that a proportion situation exists when the comparison is multiplicative.

Multiplicative structures

Vergnaud (1983) described how proportional reasoning is to work in two 'measure-spaces', which he labelled as M_1 and M_2. Measure spaces basically refer to the two components of comparison in a proportion situation. For example, when comparing the ratio of cordial to water in fruit juice mixes, or considering a rate such as the cost of petrol per litre, two components are being compared. When dealing with the quantities of the components, two types of analysis can occur: a 'between' analysis and a 'within' analysis. Consider the following example.

> A cordial drink is made up of a mixture of 1 part cordial to 5 parts water. How much water needs to be mixed with 40 millilitres of cordial to make a drink of the required strength?

The two measure-spaces (M_1 and M_2) are the cordial and the water respectively. Considering the situation from within the spaces, the new cordial amount is 40 times the original amount. To solve the proportion problem, use of the within strategy involves thinking that in the first space the quantity of cordial is multiplied by forty. Therefore, in the second measure space the quantity of water must also be multiplied by forty to maintain the relationship. Considering the situation between the two measure-spaces, the amount of water (M_2) is five times that of the cordial (M_1). Therefore, to find how much water is needed when there is 40 millilitres of cordial, this amount is multiplied by five. The multiplicative basis of proportional thinking in the two measure spaces is apparent.

In describing why proportional reasoning is a stumbling block to many students in the middle years of schooling, Behr et al. (1992, 300) stated that "the elementary curriculum is deficient by failing to include the basic concepts and principles relating to multiplicative structures necessary for later learning in intermediate grades." The scope and breadth of prerequisite knowledge for proportional reasoning was enunciated by Post, Behr and Lesh (1988, 80), who stated that proportional reasoning:

> ... requires a firm grasp of various rational number concepts such as order and equivalence, the relationship between the unit and its parts, the meaning and interpretation of ratio, and issues dealing with division, especially as it relates to dividing smaller numbers by larger ones.

Other authors have provided further opinions. English and Halford (1995, 254) stated that "fractions are the building blocks of proportion." Behr et al. (1992, 316) argued that "the concept of fraction order and equivalence and proportionality are one component of this very significant and global mathematical concept." Behr et al. (1992) further stated that the development of an understanding of ratio and proportion is intertwined with many mathematical concepts, including multiplication, division, fractions and decimals, but the essence of proportional reasoning lies in understanding the multiplicative structure of proportional situations.

Vergnaud (1983, 127) argued against instruction that separated mathematical topics that share the multiplicative structure, such as multiplication, division, ratio and linear function, pointing out that they are "not mathematically independent of one another, and they are all present in the very first problems that students meet." It would appear that multiplicative thinking is promoted through assisting students to make connections between various mathematical topics. Using Vergnaud's measure space representation of proportion situations, computational connections can be made. An understanding of division as the inverse of multiplication is also required in order for students to operate successfully in the proportion domain. Consider the following example:

> A bulk pack of sweets contains 1.5 kg and costs \$3.60. If the sweets are repackaged into 250 g packs, what price should each pack be to break even?

Using a within strategy, the second quantity is one-sixth the first. Therefore, the cost for 250 g of sweets will be $\$3.60 \div 6 = \$0.60$.

Computational links in proportion topics

When using the measure-space representation for proportion problems, typically three elements are given and the task is to find the fourth element. Such proportional problems are generally referred to as "missing value" problems. Students in the middle years have often been taught to solve such problems by writing an algebraic equation to represent the problem and then solving the equation. The standard solution procedure for solving proportion equations is via algebraic means: "cross-multiply and solve for *x*" (Post, Behr and Lesh 1988, 81). The teaching of the standard algorithm continues to be a controversial issue. For example, Hart (1981, 101) stated, "Teaching an algorithm such as $^a/_b = ^c/_d$ is of little value unless the child understands the need for it and is capable of using it." The main criticism in 'equationising' proportion situations is that the focus moves to equation solving rather than thinking in terms of the proportional nature of the problem situation. The proportion equation also uses fraction notation, although the proportion situation is rarely fractional in the sense of being a part to whole multiplicative comparison.

School mathematics textbooks

In Australian secondary schools, as in many parts of the world, students have their own copies of a school-selected mathematics textbook. The ways teachers relate to the textbook and use it in implementing the curriculum varies. Remillard (2000, 336) described the ways two teachers in her study read the textbook: "Their reading was selective and interpretative. They read different parts of the text and drew on their own perceptions to make meaning of what they read." However, recent research has provided evidence that the majority of mathematics teachers in secondary schools generally follow the prescribed textbook when planning and implementing their program (Thompson and Fleming 2004). Even if teachers' reading is interpretive, by the nature of the presentation of topics and exercises, textbooks provide a suggested sequence for planning instruction. In fact, textbooks have been described as the 'de facto curriculum' (Budiansky 2001).

In a comparative study of Grade 8 textbooks from a range of European, Asian and North American countries, Howson (2005, 19) observed that most school mathematics textbooks were now "unified texts" that include all the mathematical topics needed for a course, rather than the separate algebra and geometry texts used in the past. The chapters in these unified texts were presented in "what too frequently appears a random manner", with no overall structure and links between topics. Howson focused part of his study on the ways the different texts presented the topic of directed numbers and reported on the similarities and differences. While many of the books connected directed number ideas with real-world situations such as temperature, some adopted a formal algebraic approach to the topic. Fan and Zhu (2007) compared lower secondary grade level junior high school mathematics textbooks from China, Singapore and the United States. Their focus was on the ways the textbooks assisted students in the development of problem-solving procedures. The textbooks all demonstrated some general problem-solving procedures and heuristics, but this aspect was much more explicit in the Chinese textbooks. Interestingly, Fan and Zhu (2007, 72) observed "that there existed considerable gaps between national syllabuses/curriculum standards and the textbooks developed following on these documents."

As textbooks typically provide a sequence of material, suggest the content to be taught, and provide activities and instructional ideas for engaging students (Reys 2004), they are a valuable resource for many teachers. Rather than merely being regarded as a means of maintaining and controlling the curriculum, textbooks have been identified as potential agents of change to transform the curriculum (Collopy 2003; Grant et al. 2002; Remillard 2000). This, of course, would depend upon the extent to which the textbooks were aligned to relevant syllabus documents and educational reform agendas.

Reys (2004) has argued the need for wise selection of textbooks to support the development of students' mathematics learning and attainment of learning outcomes. To support teachers in selecting quality support materials and resources for mathematics teaching, Project 2061 (Kulm, Morris and Grier 2000, 1) developed a procedure for analysing mathematics curriculum materials, including mathematics textbooks. Recognising that curriculum materials are expensive, and that adopting particular curriculum materials will influence instruction for several years after their purchase, Kulm et al. state that curriculum materials must be evaluated to determine "their effectiveness in helping students to achieve important mathematical learning goals for which there is broad national consensus."

Methodology

The Project 2061 procedure for analysing curriculum materials begins with core mathematics Standards, or Benchmark statements, progressing to procedures for analysis of materials in the light of the targeted Standards, and against a background of core literature on effective teaching and learning of mathematics. In the study reported here, we adapted the Project 2061 procedure, using a slightly different order. First, drawing upon our analysis of the literature, we identified a set of four core learning outcomes for ratio and proportion relating to additive versus multiplicative comparison, multiplicative links across proportion situations, representation of proportion situations and links to fraction ideas. We devised indicators to elaborate the core learning outcomes, or specific curriculum content goals (SCCG), as follows:

1. *Additive and multiplicative comparison is contextualised*. Indicators: (a) real-world examples of both additive and multiplicative comparisons are given; (b) opportunities to differentiate additive and multiplicative comparisons are provided; (c) conceptualisation of situations as either additive or multiplicative is encouraged through a range of authentic real-world contexts for comparison.
2. *Common multiplicative structures and proportional thinking highlighted*. Indicators: (a) comparative relationship of proportional situations is clearly defined; (b) common multiplicative structure is made explicit in different problem types (such as ratio, rate, scale, chance, gradient, and the like); (c) the use of proportional thinking to solve problems is made explicit in worked examples.
3. *Effective use of a range of representations*. Indicators: (a) tables, linear graphs and number lines are used to represent proportional situations;

(b) representations build understanding of the multiplicative structure of the problem types; (c) applications are genuine use of the appropriate concepts in the students' environments.

4. *Related fraction ideas are explicitly connected.* Indicators: (a) clear links are made with ideas of fraction and equivalence; (b) identification of part-whole fraction and part-part-whole ratio relationships is explicitly distinguished; (c) the introduction of the formal "proportion equation" is delayed until extensive experience has been gained with other representations.

Second, the principles underlying the mathematics syllabus (Queensland Studies Authority 2004) for the Australian state in which this study was conducted were used as a backdrop for analysis of the curriculum materials in terms of their capacity to promote students' attainment of core learning outcomes of a particular topic, which in this case was ratio and proportion. Our analysis of the syllabus revealed strong alignment to the principles of equity, curriculum, teaching, learning, assessment and technology as proposed by the National Council of Teachers of Mathematics (NCTM 2000). The Mathematics Principles are outlined below.

Equity – high expectations and strong support for all learners of mathematics through development of skills, knowledge and positive dispositions, as well as an appreciation of the cultural, historical and economic contexts of the development of mathematics knowledge;
Curriculum – interconnected and coherent;
Teaching – providing experiences for making sense of life experiences and solving problems;
Learning – developing deep understanding through active engagement in mathematical investigations, communicating thinking and reasoning;
Assessment – opportunities to demonstrate their knowledge in a variety of ways;
Technology – integral to everything that students do.

Using the SCCG and Mathematics Principles, the analysis occurred at three levels. At the first level, we overviewed the scope, sequence and presentation of material on ratio and proportion and related topics. At the second and third level, we rated the materials as either high, medium or low against the four SCCG and the Principles respectively. In this study, we analysed two eighth-grade textbooks (Silva 2003; Brodie and Swift 2006), selected because the series from which they come have a long tradition of use in mathematics classrooms in the state of Queensland. Both series have recently been revised and reprinted, updated to align with the new mathematics syllabus in the state. In this report, we refer to these two textbooks as Text 1 and Text 2 respectively.

Results

Overview of textbooks

An overview of material related to proportional reasoning within each of the textbooks was compiled, using a summary sheet to record chapter titles, subheadings, page count, presentation of material, and additional features. The completed summary sheet is presented in Appendix 1. In general, both texts provide worked examples followed by exercises. Both texts have pictures and diagrams interspersed that serve to contribute to an 'uncluttered' feel. As shown in Appendix 1, Text 1 contains more additional features than Text 2, with a chapter review test as well as an investigative assessment task, several suggested investigations (although not always related to the topic of ratio and proportion), and a pre-chapter diagnostic test with practice worksheets contained on an accompanying CD Rom. Also, in Text 1, each subsection of the chapter presents one worked example with further explanatory text, followed by practice exercises and word problems, whereas Text 2 contains several worked examples (eight in total under the subheading Ratios and Rates) before any practice exercises are given. A further noted feature of Text 2 is that all presented examples are consecutively numbered from the beginning of each chapter. Hence, for the Fractions and Ratios chapter, a total of 25 examples are given, 11 of which relate specifically to ratio (numbered from 15 to 25).

Specific Curriculum Content Goals

Taking each of the SCCG in turn, we analysed material in each textbook to rate the degree (high, medium, or low) to which each SCCG was addressed. Our rating for each textbook against each of the SCCG is presented in Table 1. Specific examples of material from each of the textbooks are then presented to support each rating.

The first SCCG relates to contextualisation of the additive and multiplicative field through the presentation of real-world examples of both additive and multiplicative comparisons, and the provision of opportunities for students to differentiate and conceptualise real-world additive and multiplicative situations. In our reading of both texts, we could not find any discussion or examples of additive comparisons, and no examples were given of situations in which students were required to recognise or distinguish between the two types of comparison. In Text 1, ratio is defined as "a comparison between two amounts using numbers" (Text 1, 171). Early ratio exercises are devoted to situations being expressed as ratios, including exercises given in pictorial form (a diagram of 3 people and 7 dogs on leashes); numerical form (12 litres to 13 litres); and contextual form ("Bill is a pigeon fancier. His total of 43 pigeons includes 17 white, 19 speckled and 7 grey. Write the ratio of . . ."). The purpose of such exercises appears to be symbolic representation of ratio situations. In Text 2, the topic of rate is addressed first in the chapter with a real-world situation (cost of tomatoes per kilogram). A definition of rate is given: ". . . a rate compares related measurements. The measurements that are compared within a rate are usually referred to as 'quantities'" (Text 2, 177). The word ratio is defined as follows: "When you compare quantities of the same kind, you obtain a

Table 1. Rating of each textbook on the Specific Curriculum Content Goals (SCCG).

SCCG	Text 1	Text 2
1. Additive and multiplicative comparison contextualised	Low	Low
2. Common multiplicative structures and proportional reasoning highlighted	Low	Medium
3. Effective range of representations	Low	Medium
4. Related fraction ideas explicitly connected	Low	Low

ratio. A **ratio** compares quantities of the *same kind* in a definite order" (Text 2, 178) [emphases in original]. Eight worked examples follow, and although the practice examples are couched in real-world contexts and are multiplicative, there are no additive real-world situations given, and no opportunity for students to differentiate additive and multiplicative comparisons. We rated both texts as low on SCCG 1: Additive and multiplicative comparison is contextualised.

The second SCCG relates to the degree to which common multiplicative structures and proportional thinking are highlighted and how this structure is connected across different problem types (e.g., scale, ratio, rate); and the extent to which worked examples promote proportional thinking. Both textbooks present ratio (and in the case of Text 2, rate) examples that demonstrate multiplication and division procedures. However, while the problems are all structurally similar, the solution method for each one appears to be dictated by the problem situation or context, rather than the actual problem type. For example, in Text 2, the eight worked examples all present a different solution procedure, each with its own set of steps, with no explicit connections being made between the different problems. The practice exercises for students are worded in a similar fashion to the given examples, suggesting a strategy of matching the task to a solution procedure demonstrated in the worked examples.

In terms of how the textbooks make explicit the common multiplicative structure across proportional topics (e.g., scale, rate, gradient), Text 1 includes a subsection on scale factors. The worked examples in this subsection make no reference to previous procedures. Text 2, where ratio and rate are presented together, has this potential, but it is not realised. This textbook attempts to link ratio and rate:

> Rates and ratios are closely related. For example, a speed of 5 m/s means that the object travels 20 m in 4 s and 30 m in 6 s. The ratio of the distances, 20:30, is the same as the ratio of the times, 4:6, because they both simplify to give 2:3. (Text 2, 179).

Subsequent exercises do not directly show this connection in practice. No worked examples draw similarities in procedures for rate or ratio problems. In this textbook, ratio and proportion concepts are met in other chapters and sections. *Scale drawings and maps* precedes the chapter on fractions and ratios, and the term "scale factor" is used. In a later chapter on *Similarity and scale*, students are introduced to enlargements and reductions as follows: "Enlargements and reductions produce an image that is a different size from the original. The amount by which the shape is enlarged or reduced is called the magnification" (Text 2, 211). The terms ratio and scale factor are not mentioned in this section, with "scale factor" being replaced by "magnification". It was noted that the definition given did not mention multiplication. This text provides a greater range of topics that are underpinned by proportional reasoning than Text 1, yet its presentation in no way explicates the multiplicative nature that links the topics. We rated Text 1 as low and Text 2 as medium on SCCG 2: Common multiplicative structures and proportional thinking highlighted, due primarily to the greater breadth of proportional reasoning situations given in Text 2.

The third SCCG relates to the effective use of a range of representations, including the degree to which graphs, tables, and other diagrams are used to promote conceptual understanding of routines and rules, as well as the extent to which given examples are authentic. The use of a table is a suggested means of explicating the multiplicative structure of ratio situations (e.g., English and Halford 1995; Hart 1981; Robinson 1981). Text 1 provides a table of results for mixing red and blue paint in the correct ratio of 3:2 to find how much blue paint is required for 21 litres of red paint. The worked solution presents a series of multiples of the ratio 3:2 (6:4, 9:6, and so on) to find the missing number in 21:?. Rather than analysing the table to discuss the change, the instructions for solution are simply "multiply 3 by 7 to get 21, the 2 by 7 to get 14" (Text 1, 179). A further worked example in another subsection of Text 1 suggests the construction of a table to organise the information. For example, to determine the length of Rex's fish compared to Edna's when the ratio is 5:6 and Rex's fish is 34 cm long, the following steps are listed:

> (1) Put the values in a table, with headings (Let *f* represent the length of Edna's fish); (2) Write the equation with *f* on the left-hand side; (3) Invert both fractions; (4) Solve for *f*; (5) Write the answer in words (Text 1, 184).

The worked example includes the table to show how to arrange the information.

	Ratio parts	*Length (cm)*
Rex	5	34
Edna	6	*f*

Even though the table arranges the pieces of information to assist in construction of the proportion equation, the multiplicative relationship is not overtly emphasised, and between and within analysis is not suggested. Further, the use of the table seems to be quite redundant when a previous example involving numbers without a context (Find y if $3{:}y = 5{:}7$) produces the same format from following the steps: (1) Write the ratios as fractions, (2) Invert both fractions, (3) Solve for y:

$$\frac{3}{y} = \frac{5}{7} \qquad \frac{y}{3} = \frac{7}{5} \qquad y = \frac{7}{5} \times 3 \qquad y = 4.2 \ \ \text{(Text 2, 184)}$$

In the Direct Proportion subsection of Text 2, a table is used to represent a rate situation, and a linear graph is used to represent a speed (distance-time) problem, with practice exercises requiring students to complete tables and in some cases, draw graphs, yet analysis of the tables and/or graphs is not suggested as a means of predicting and describing the comparisons being represented in these modes.

In terms of authentic genuine problems, both textbooks include word problems that appear to be an attempt to link the material with real-world situations, e.g., "Sally made punch for her party. The punch had 3 parts pineapple juice to every two parts of lemonade. She made up 25 litres of punch altogether. What volume of each ingredient was required?" (Text 2, 183). However, some appear to be 'un-real' applications:

> A teacher does an analysis of the time she spends on teaching and the time she spends on other duties (such as preparation and correction), and finds that those times are in the ratio 5:3. If she taught 5 periods of length 50 minutes one day, estimate the time she spent on other duties. (Text 1, 187)

We rated Text 1 as low and Text 2 as medium on SCCG 3: Effective use of a range of representations, primarily due to the greater range of representations in Text 2 and more realistic real-world problems.

The fourth SCCG relates to connecting and distinguishing fraction part/whole concepts and ratio part/part/whole concepts and relationships. Early in Text 1, ratios are linked to fractions as a permissible way of representing ratios:

> Ratios may [also] be written as fractions.
>
> For example, the ratio 1:5 may also be written as $\frac{1}{5}$
>
> the ratio 7:3 may be written $\frac{7}{3}$ (Text 1, 171)

The link with fractions is made again in the discussion of equivalent ratios: "... ratios can be cancelled down or simplified just like fractions, by dividing by the highest common factor"(Text 1, 174). Diagrams with rectangles partitioned into equal areas are used to demonstrate equivalence, expressing the ratio of the shaded area to unshaded area. In Text 2, fraction representation is also presented early, with a subtle suggestion that fraction means division:

> A rate is obtained by dividing one quantity by a different, related quantity. It is sometimes useful to express a rate as a fraction. (Text 2, 177)

Ratio and fraction are further linked in this textbook (Text 2, 178) with the reader being informed that fraction notation can express a ratio "with the second quantity as the denominator." From the worked rate and ratio examples given, the procedures are the same: write the division (for rate problems) or as a fraction (for ratio problems), cancel by common factors (simplest form ratio), or use a calculator (if there are no common factors). In both textbooks, the fraction format is presented, and fraction rules (simplest form, common denominator, division) are applied. Clearly, by converting ratios and rates to fractions, there appears to be an expectation from the textbook writers that students will abandon their part/whole knowledge of fractions at this time and operate on the proportional reasoning tasks at a purely mechanical level.

In both textbooks, some links to fraction language are made when looking at 'sharing' problems, that is, sharing something in a given ratio. In Text 1, the first step in the presented method is to add the numbers in the ratio to find the total number of parts, and find the size of one part. Text 1 steps the method as follows.

1. Add the number of 'parts' in the given ratio.
2. Divide the amount by the number of parts to find the amount per part.
3. Multiply each value in the ratio by the amount per part.
4. Check that the new amounts add up to the original amount. (Text 1, 198)

The method is specified as a procedure, and no other discussion is provided to make possible links between this method and the part/whole notion of fraction, nor is there any mention of the proportional nature of this situation.

In Text 2, for sharing problems, the words 'part' and 'total' are explicitly used. In the problem, "Jan and Dean are to divide $46 in the ratio of 2:3. How much does each get?", the following solution procedure is provided (Text 2, 180):

Total parts $=2+3=5$			
Value of one part $=\$46 \div 5=\$9.20$			
Jan's share	$=2$ parts	Dean's share	$=3$ parts
	$=2\times\$9.20$		$=3\times\$9.20$
	$=\$18.40$		$=\$27.60$
Total amount	$=\$18.40+\$27.60$		
	$=\$46$ ✓ OK		
Jan gets \$18.40 and Dean gets \$27.60			

In both textbooks, it is in the context of sharing problems involving ratio that the part/part/whole structure is evoked, yet not in a part/whole fraction sense. We rated both Text 1 and Text 2 low on SCCG 4: Related fraction ideas explicitly connected.

Alignment with the Mathematics Principles

Taking each of the Mathematics Principles in turn, we analysed material in each textbook to rate the degree (high, medium, or low) to which we felt the material aligned with the Mathematics Principles. Although the Mathematics Principles were addressed to varying degrees in each textbook, we rated both texts as low on each of the principles, as shown in Table 2. Our reasoning for each rating follows.

The Equity Principle relates to high expectations, developing skills, knowledge and building positive dispositions to the study of mathematics, and awareness of the cultural contexts from which mathematics knowledge derives. In Text 1, investigations are provided (the golden rectangle, ancient multiplication methods), thus providing historical links to mathematics topics. Text 2 suggests some specific ratio-related tasks. Both texts provide many practice examples, thus catering for faster and slower workers as required. The format of both texts follows a fairly typical textbook layout, and thus suggests a traditional approach to teaching mathematics (demonstration of procedure, practice of procedure). With little active investigation, such approaches are not typically regarded as ones that would support the development of positive dispositions to the learning of mathematics.

The Curriculum Principle relates to a connected and coherent curriculum and the degree to which students are supported to see links between big mathematical ideas. Whilst the two texts have organised some proportion-related topics together, the range of solution procedures for solving proportion problems potentially masks the connections between the topics. The presentation of the material, particularly in Text 2, provides a very disjointed approach. In this text, students are introduced to eleven worked examples of ratio and rate situations, all with different solution procedures,

Table 2. Rating of each textbook on the Mathematics Principles.

Mathematics Principles	Text 1	Text 2
Equity	Low	Low
Curriculum	Low	Low
Teaching	Low	Low
Learning	Low	Low
Assessment	Low	Low
Technology	Low	Low

followed by a range of mixed exercises. Students would have some difficulty in consolidating understanding of the procedures or of building conceptual knowledge of ratio and proportion. Text 1 presents exercises in a way that supports mastery of particular skills before introducing new skills, but does not make explicit the links between exercises and procedures. Both texts have avoided opportunities to make connections by changing the language used. The omission of the word 'rate' completely from Text 1 and the change from the term 'scale factor' to the term 'magnification' in Text 2 are just two examples.

The Teaching Principle relates to problem solving and real-world investigations. Both texts provide a range of real-world examples, although examples appearing in Text 2 are somewhat more plausible than those in Text 1. In Text 2, the context appears to be a cue for the suggested solution procedure. For example, when discussing speed situations, the instructions are given: Use rate = distance/time and then use the rate relationship *Distance = Rate x Time* (Text 2, 185). For an exercise on scale drawing, the rate equation is to be applied: *Drawing = Scale x Real Size* (Text 2, 186). The contexts appear to dictate the procedures. These two examples further highlight how connections between big ideas in ratio and proportion situations are not suggested (the Curriculum Principle). In Text 1, the presentation of real-world situations appears to be dictated, to some extent by the procedures to be practised by the students. For example, Text 1 (190) includes a procedure for increasing or decreasing a number by a given ratio, with the steps summarised as follows: "To *increase* an amount in a given ratio you multiply by an *improper fraction* formed from the values in the ratio and to *decrease* an amount you multiply by a *simple fraction*" (emphases in original). Upon further analysis of this text, we found two word problems to which the above methods could be applied, yet the real-world nature of these situations is unlikely:

> Sam buys a car for $30 000 and when he drives it out of the showroom its value decreases in the ratio 5:6. Find the decreased value of the car.
>
> The amount of guano mined on a remote island decreased one year from 6900 tonnes in the ratio of 20:30. Calculate the reduced amount of guano mined. (Text 1, 191)

With the range of proportional contexts given, the appropriateness of an additive comparison over a multiplicative comparison is not addressed.

The Learning Principle relates to developing deep knowledge. Again, through over-emphasis on procedures rather than providing reasons for the procedures, students are offered little assistance to develop deep understanding of the similarities and subtle differences amongst proportion-related topics as highlighted through discussion of the Curriculum and Learning Principles above. One of the main omissions from both texts is distinguishing ratios from fractions, or explaining why fraction representation holds for ratio situations. Both texts also omit discussion of both additive and multiplicative comparisons. Ratio and proportion requires students to expand their knowledge and understanding of fractions. Fraction procedures are accompanied by the statements: "ratios may also be written as fractions" (Text 1, 171) and "It is most common to write ratios in simplest form using whole numbers, in the same way that we write fractions in simplest form" (Text 2, 178). The material in Text 2 is contained within the chapter entitled *Fractions and Ratios*, yet there are no conceptual links highlighted, only procedural links. Text 1 presents three congruent rectangles divided into 7, 14, and 21 parts respectively, suggesting that such diagrams "may be familiar from your study of fractions" (Text 1, 174). The diagrams are not shaded, but the given information states that equivalent ratios of 1:6, 2:12 are similar to the proportion of shaded and unshaded area. Students are then informed "we don't need to look at the diagrams to know these ratios are equivalent – ratios can be cancelled down just like fractions."

The Assessment Principle relates to a range of ways to demonstrate mathematical understanding. Both texts provide chapter tests as well as a glossary of terms for revision. Text 1 provides an assignment task, which, by its inclusion, suggests an alternative form of assessment. Text 2 contains some investigative tasks for interest. Due to the structure of both texts, there would potentially be little time to complete all tasks and develop proficiency in solving all types of problems. The time to enable students to undertake the suggested investigations may possibly be beyond instructional time available.

In terms of the Technology, neither text proposes the use of technology to build conceptual understanding. Both texts, however, include a CD ROM. While these contain prepared spreadsheets for some activities, the material contained on the CD is predominantly aimed at further consolidation of skills and procedures.

Discussion

The textbook examination was based on a set of Specific Curriculum Content Goals devised from our analysis of literature on proportional reasoning, and Mathematics Principles based on relevant curriculum documents. The three steps in this examination enabled us to present an overview of the structure, sequence and features of the textbooks' material relating to ratio and proportion, and rate the extent to which they aligned with our SCCGs and the Mathematics Principles respectively. Had different criteria been devised, quite different results would have been attained. However, the two textbooks in question were published to meet the requirements of a particular syllabus and the Mathematics Principles were derived from that syllabus document. Further, rating of each text according to the criteria is not quantified although coupled with specific examples of text material to support the given rating. Hence, this is an acknowledged limitation of the analysis reported in this paper.

Against the SCCG, the two textbooks analysed here appear only minimally to promote proportional reasoning and multiplicative thinking. Both texts present many real-world applications of ratio as a proportion through worked examples and practice exercises and tasks, yet the nature of ratio and proportion as a multiplicative comparison is not overtly addressed. The multiplicative structure in proportional situations is not explored, masked by myriad procedures for different problem situations. Proportional reasoning is about knowing proportion as a comparison in multiplicative terms (Behr et al. 1992). Neither textbook provides opportunities for students to make change comparisons in additive and multiplicative terms, and hence the potential of the study of ratio to enhance proportional reasoning is not taken up. It is through such interpretations of change situations that understanding of fraction and ratio equivalence is promoted (Behr et al. 1992). In both textbooks, fractions are used to represent ratios, yet the part/whole fraction concept and part/part/whole ratio concept is not flagged. Equivalent fraction procedures are advocated for equivalent ratio manipulation. Using fraction understanding is a key aspect of proportional reasoning (English and Halford 1995) which is more than computational procedures.

The textbook analysis highlighted the range of computational procedures presented to students through their study of ratio and proportion, and also made apparent the minimal use of diagrams, tables and graphs to further proportional reasoning. It was also seen that the symbolic representation of proportional situations $^a/_b = {^c/_d}$ does not feature strongly, although it was implied in some given examples. Certainly, the cross-multiply procedure is not an advocated solution method in either text. However, through presentation of many procedures, it seems that students would be allowed little time to become familiar with the standard proportion representation and its inherent meaning. Proportional reasoning is the capacity to work in two measure spaces (Vergnaud 1983). Through analysis of how elements within the proportion equation relate to each other, there is greater capacity for students to build a holistic conceptualisation of ratio and proportion. In these two textbooks, students' capacity to make connections between the elements in the proportion situations is blurred through the listing of steps for solution in every example given. In Text 2, specifically, there are 11 explicitly identified different procedures for the topic of ratio and rate.

Although this analysis has highlighted some of the textbooks' limitations, the value of the textbook as a teaching resource must be considered. A textbook provides a range of graded tasks and exercises designed to orientate students to the conceptual field of the topic which they are studying, and thus provides teachers with material upon which they can draw to build into their teaching. Textbooks are a resource to support the teaching and learning of mathematics, and also have the capacity to promote pedagogic and curriculum reform.

Concluding comments

In this study we adapted an earlier method of evaluating mathematics materials (Kulm et al. 2000) so that it reflected the requirements of a specific mathematics syllabus as well as reflecting research literature on the teaching and learning of a specific mathematical topic. The development of the criteria was a challenging task that involved careful examination of relevant curriculum documents as well as a detailed survey of literature focused on the chosen topic area. The criteria served to direct our exploration and analysis of the capacity of two contemporary textbooks to mediate students' proportional reasoning development. The textbooks examined in the study were found to be quite limited in the ways they addressed the requirements of a syllabus that stresses a coherent approach to the teaching and learning of mathematical topics. The approach of both books was very much concerned with procedures for different types of problems, with no apparent attempts to highlight links between the problems types or the underlying common structure of the problems.

Although not yet trialled with practising teachers, this instrument could be a useful beginning point for teachers to make a preliminary analysis of the textbooks they currently use. Further research would determine the extent to which teachers found such an instrument useful. The process in itself would alert teachers to the importance of critically examining textbooks prescribed for students within their schools. Results of such a trial would also provide vital information about the user-friendly nature of the instrument, and possible modifications for improvement.

Kaput and West (1994, 284), over a decade ago, urged that the teaching of ratio required major attention: "Although the tradition of teaching ratio reasoning in the formal style is very long-lived, we should not assume that it should be venerated, or continued." Further, Kaput and West argued that the rush to computational procedures is the basis for students' disengagement with mathematics in the middle grades. Analysis of popular mathematics textbooks currently in use in schools is a potential means to raise awareness of instruction in key topics within the school mathematics curriculum.

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Appendix 1. Initial analysis of textbook material

Topic of focus: Proportional Reasoning
Grade Level of Textbooks: 8

	Text 1	*Text 2*
Chapter titles related to topic and page count	Ratio, 47 pages	Fractions and Ratio, 12 pages
Subsections in chapter	● Writing ratios ● Simplifying ratios ● Using ratios to find amounts ● Using ratios to find amounts – harder problems ● Changing an amount in a given ratio ● Decimal ratios ● Dividing an amount in a given ratio ● Ratio and scale drawing	● Ratios and rates ● Direct proportion
Related subsections in other chapters		● Scale drawings and maps, 7 pages (in Length and Perimeter Chapter) ● Similarity, 1 page (in Shapes and Areas Chapter)
Sequence of material in each subsection	● Definition/explanation ● Worked example ● Exercises and word problems	● Definitions and worked examples ● Exercises ● Modelling and problem solving
Additional Features		
Investigations	Yes (5) ● Egyptian Multiplication ● Russian Multiplication ● How old is your dog in human years? ● Concrete calculations ● The beautiful ratio	No
Quiz skills practice	Yes (1)	No
Assessment tasks	● Review Test ● Investigative Assessment Task (Bicycle Gears)	No
Key points summary	Yes	Yes
CD Rom	Yes, further practice worksheets	Yes, further practice exercises, syllabus check, parent guide