

Ratio Tables to promote PROPORTIONAL REASONING in the primary classroom



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describes ratio tables
and demonstrates
how they can be used
to support the
development of
proportional
reasoning.

There are many classroom activities that can be implemented with primary students that introduce and immerse them in ratio investigations. Some enjoyable tasks include finding the ratio of tongue rollers to non-tongue rollers in the classroom; left-hand dominant to right-hand dominant; right-thumb over left-thumb when clapping the hands to left-thumb over right-thumb, and so on. Children also enjoy looking at body ratios and finding that their foot is the same length as the distance from their wrist to their elbow; that their wrist circumference is half their neck circumference, which is half their head circumference (or near enough), or that their height is almost the same length as their arm-span. However, extending from these investigations to develop understanding of the multiplicative relationship between quantities in a ratio is often difficult as this requires proportional reasoning.

Proportional reasoning is being able to make comparisons between entities in multiplicative terms (Behr, Harel, Post & Lesh, 1992). This means that the relationship between the two entities is conceptualised as a multiplicative relationship. For many young children, comparisons between entities are described in additive terms, and they compare groups using additive or subtractive language. For example, when comparing the number of boys to girls when the ratio of boys to girls is 2 to 3, they may say that there is always one extra girl for each group of boys. So, if there were 4 boys, there would be five

girls. Being able to describe proportional situations using multiplicative language is an indicator of proportional reasoning.

The development of proportional reasoning is something that takes time. It is fostered by quality learning experiences in which students have opportunities to explore, discuss and experiment with proportion situations. Proportional reasoning is also dependent upon sound understandings of associated topics, particularly multiplication and division. Other mathematical activities through which proportional reasoning develops include the study of rational number topics such as fractions, decimals, percentages, scale drawing, and of course ratio and proportion. Proportional reasoning is fostered through rich conceptual understanding of ratio and proportion, but these are difficult concepts that present a challenge to many students (Ben-Chaim, Fey, Fitzgerald, Benetto & Miller, 1998; Lo & Watanabe, 1997).

Ratio tables

One way of assisting students to develop mental strategies for solving proportion problems is through the use of ratio tables (Middleton & van den Heuvel-Panhuizen, 1995). Ratio tables are a convenient way of symbolising the elements within proportion situations, and for supporting thinking strategies for solutions. Ratio tables encourage the use of number strategies such as halving, doubling, multiplying by 10, and so on.

A ratio table is a tool that assists in looking at the relationship between two quantities. The table is constructed to show the two quantities and their values. Progressive and simultaneous operation on the given numbers shows how the relationship (ratio) is preserved proportionately. The following example shows how a ratio table is used to calculate how many rabbits can be housed in a given number of enclosures, when each enclosure can house 12 rabbits.

EXAMPLE 1

**Each enclosure can hold 12 rabbits.
How many rabbits can fit in 14 enclosures?**

STEP 1: *Construct the ratio table and display the given information.*

Enclosures	1				
Rabbits	12				

STEP 2: *Select successive operations (e.g., multiplying by 10, halving, doubling) to determine the solution. Use arrows above each number to show the journey to the solution.*

		$\times 10$	$\div 2$	$\times 3$	-1
Enclosures	1	10	5	15	14
Rabbits	12	120	60	180	168

Figure 1. Using a ratio table to solve the rabbit enclosure problem.

In this example, the pathway to the solution was to multiply by 10, then divide by 2, then multiply by 3 to get to 15. This is one more than the required number, so then just subtract 1 group of rabbits (12) to reach the solution of 168.

These two examples show different pathways that can be taken to arrive at the solution. In the first example, after multiplying by 10 and then halving, the next step was to multiply by 3. In the second example, after the amount for 10 and 5 had been determined, these two amounts were added. This required looking back at the table to locate previous calculations that assisted in reaching the destination.

EXAMPLE 2

Chocolate bars are packaged into 15 bar boxes. How many chocolate bars would there be in 16 boxes?

STEP 1: *Construct the ratio table and display the given information.*

Boxes	1				
Choc Bars	15				

STEP 2: *Select successive operations (e.g., multiplying by 10, halving, doubling) to determine the solution. Use arrows above each number to show the journey to the solution.*

		$\times 10$	$\div 2$	$+(10+5)$	$+1$
		\curvearrowright	\curvearrowright	\curvearrowright	\curvearrowright
Boxes	1	10	5	15	16
Choc Bars	15	150	75	225	240
		\curvearrowleft	\curvearrowleft	\curvearrowleft	\curvearrowleft

Figure 2. Using a ratio table to solve the chocolate bar problem.

In this example, the pathway to the solution was to multiply by 10, then divide by 2, then add 10 and 5 to get to 15. This is one less than the required number, so then just add 1 package of chocolate bars (15) to reach the solution of 240.

To use ratio tables effectively, students need lots of practice. There are several issues about ratio tables that actually inhibit students from using them as a tool, as follows:

1. Ratio tables are time-consuming to construct. In order to overcome this, neatness may have to be sacrificed so that students' focus is not on ruling lines and constructing regular columns, but is

directed to number patterns and exploring relationships in the situation.

2. Ratio tables can be extended infinitely until a satisfactory destination is reached. In some cases, students' feel reluctant to extend the ratio table to add more cells. Conversely, they may feel that they need to continue putting numbers in each cell to fill up empty cells. Students need to be continually reminded that the table is a tool to help them arrive at a solution and that they are in control of constructing and using the table.
3. Ratio tables are meant to show the sequence of a user's calculations, not show ordered calculations from smallest to largest. For example, in the chocolate bars ratio, some students would find it difficult to record the "5" after the "10" because "it is out of order." Students need to be reassured constantly that their record of calculations does not need to be ordered from smallest to largest.
4. Following on from point 3, students need constantly to be reminded to check that each progressive calculation matches the given ratio. This reinforces the multiplicative relationship within the situation and thus promotes proportional reasoning.

Promoting ratio tables as a tool

Children will benefit from being given opportunities to explore completed ratio tables that show different solution pathways to the same problem, as in the two examples shown in Figure 3. The task is for the students to include the arrows to show the solution journey.

Sarah and Simon were working out the solution to the following problem: *Seedling plants come in boxes of 35 plants. How many plants would be in 16 boxes?* Determine each child's solution strategy by inserting arrows to show their calculations.

Sarah's solution strategy

Boxes	1	2	4	8	16
Plants	35	70	140	280	560

Simon's solution strategy

Boxes	1	10	2	6	16
Plants	35	350	70	210	560

Figure 3. Solutions to the seedling box problem.

Discussion of the solution strategies and other possible pathways assists students in seeing that ratio tables are tools for determining proportional situations. From the ratio table, students can be guided to explore number patterns and relationships that are occurring in the table, and to consider why the relationship between the quantities is multiplicative. Using realistic examples (packaging) provides further support for promoting understanding of the multiplicative connection in ratio and proportion situations. Further discussion and exploration of ratio tables and their solutions will gradually lead students to realise that the solution can be obtained in one step using multiplication. For example, to solve the packaging pots exercise, multiplying 16 by 35 will provide the solution. This is an important milestone in children's proportional reasoning and builds upon students' concept of multiplication. This should not, however, be rushed because the solution process would require either pencil and paper calculation or the use of a calculator unless students can perform this multiplication mentally. The value of the ratio table is that it shows the linear nature of the relationship in proportional situations that can be demonstrated through ordering the values (lowest to highest) in a ratio table.

Ratio tables for classroom investigations

Ben, a teacher of Year 5, showed his class a selection of wheels from various bikes, trikes, toy trucks, carts, vehicles. He posed the problem shown in Figure 4.

THE TRAVELLING WHEEL

How often will a wheel rotate when travelling a distance of 100 m?

1. Name each of the wheels that you will measure.
2. Measure the height of each wheel.
3. Predict how many rotations you think it will make in 100 m.
4. Measure the distance it rolls in one rotation.
5. Use a ratio table to predict how many times it will roll in 100 m.

Can you use your ratio tables to predict how many times a wheel would rotate over a distance of 170 m?

Ratio table:

Number of Rotations	1				
Distance Travelled					

Figure 4. Travelling wheel problem

Ben's students selected a wheel from the collection and set about trying to measure the distance in one rotation. The ratio table assisted in organising the data collected and in determining a solution to the problem. As each group was working with a different wheel, a range of solutions was determined. This added to the vitality of the whole class discussion as each group reported their findings. Students readily verbalised that the smaller the wheel the larger the measure; a

fundamental measurement principle. Each group also shared the calculations on their ratio table and discussed the pathway to their solution strategy.

Summary

Many mathematical tasks and activities require proportional reasoning. Drawing a plan view of a house, a “mud map” of the path from home to school, and a plan of the school yard; sharing four pizzas among three people or two chocolate bars between three people; determining the better buy when 1 kg costs \$3.50 and 1.5 kg costs \$4.20; determining whether there is more chance of selecting red from a collection of 3 red and 4 blue compared to a collection of 6 red and 8 blue, all require proportional reasoning. The development of proportional reasoning is a gradual process, underpinned by increasingly sophisticated multiplicative thinking and the ability to compare two quantities in relative (multiplicative) rather than absolute (additive) terms. Proportional reasoning as part of the multiplicative field has been identified as a key concept underlying a wide range of topics studied at the middle school level. The task for middle primary teachers is

to assist students to build, consolidate and link their proportional reasoning ability; not an easy task, as research consistently indicates students’ difficulty with proportion related topics. Ratio tables are a useful way for helping children engage in proportional situations

References

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