Fractions
Decimals and Percentages
Module 5

FRACTIONS & DECIMALS

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OVERVIEW OF MODULE 5

INTRODUCTION
This module is the fifth in a series of six that comprise a resource of activities for developing students’ mental computation in Years 3 to 10. The focus of this module is on mental computation of common and decimal fractions.

LANGUAGE
The terms common fraction and decimal fraction are used to distinguish between the symbolic representation of numbers written in fraction form (e.g., ¼) and in decimal form (e.g., 0.25) respectively. For simplification, in this module, common fractions are referred to as fractions and decimal fractions are referred to as decimals.

TEACHING SEQUENCE
The Activities presented in this module are based on two major principles for developing mental computation skills with fractions and decimals:
1. Building conceptual understanding to make calculation meaningful; and
2. Encouraging the use of mental images to assist in mental computation.

Rather than being a collection of ideas, strategies and games for mental computation, each Activity is a sequential teaching episode to develop a particular aspect of fraction/decimal knowledge and understanding for mental computation.

Each Activity is designed to stand-alone, but prior knowledge may assist students’ performance in some cases, and such prior knowledge may be promoted through a previous Activity. Collectively, the Activities are not designed as a curriculum for fractions and decimals, but rather the augment and build students’ knowledge of these topics to assist meaningful mental computation.

For each Activity, the Aim of the Activity is given, summarising the conceptual basis of the strategies presented. An Overview of the Activity is also provided, enabling the nature of the activity to be readily gleaned. Materials are indicated, with the letters BLM followed by a reference number. Black Line Masters are located at the back of the booklet. Teaching Points are presented in dot-point form for succinctness, serving as “reminder tips” that focus the key points of the Activity. Tips for assessing students’ conceptual understanding, mental computation strategies, and mental computation performance are provided under the heading of Assessing Performance. Carefully selected Practice Examples are presented to indicate the types of calculations students would be expected to perform mentally. The practice examples are aimed to serve as a guide for devising further sets of calculations for consolidation purposes.

The sequence for each teaching episode is presented on the facing page under the heading: Activity Outline. The sequence is numbered to indicate the steps along which the teaching episode proceeds.
5.1 EXPLORING UNIT FRACTIONS
(DEVELOPING PART OF A WHOLE)

AIM:
To promote part/whole conceptual understanding and to assist students perform simple fraction mental computations through visualization of a whole divided into equal parts.

OVERVIEW:
In this activity, students explore one-third through dividing various wholes into three equal parts, and then create pictures of other unit fractions.

MATERIALS:
BLM 5.1, BLM 5.2 and BLM 5.3 (one copy per student), scissors, glue.

TEACHING POINTS:
- If implemented as a whole lesson, this would require approximately 30-45 minutes
- Maintain the flow of the lesson by encouraging quick shading rather than elaborate and neat colouring

ASSESSING PROGRESS:
- Students readily discuss their visual images of fractions.
- Students discuss appropriateness of various visual images of fractions, describing how any "whole" can be divided into fraction amounts.
- Students' mental computation becomes more accurate.

PRACTICE EXAMPLES:

\[
\begin{align*}
1 - \frac{1}{3} & \quad 1 - \frac{3}{4} \\
1 - \frac{1}{6} & \quad 1 - \frac{3}{5} \\
1 - \frac{1}{10} & \quad 1 - \frac{4}{5} \\
1 - \frac{1}{4} & \quad 1 - \frac{7}{8}
\end{align*}
\]
ACTIVITY OUTLINE:

1. Present students with a series of shapes (drawn on the board) And ask them to describe how to show $\frac{1}{3}$ of each shape. Include a number line.

2. Ask key questions:
   - How many parts?
   - How many shaded?
   - What does $\frac{1}{3}$ mean?

3. Create other unit fractions with various other shapes:
   - Rectangular paper: show $\frac{1}{8}$ of a rectangle
   - Rectangular paper: show $\frac{1}{4}$ of a rectangle
   - Paper strips: show $\frac{1}{3}$
   - Paper strips: show $\frac{1}{5}$
   - Paper strips: show $\frac{1}{10}$
   - Pre-cut circles: Fold to show $\frac{1}{8}$
   - Pre-cut circles: Fold to show $\frac{1}{3}$ (see below)
   - Pre-cut circles: Fold to show $\frac{1}{12}$
   - Number lines: Show $\frac{1}{9}$
   - Number lines: Show $\frac{1}{7}$
   - Number lines: Show $\frac{1}{12}$

4. Use BLM 5.2 to consolidate. The sheet contains various wholes divided into three equal parts. Direct students to write the fraction name in words (one-third) and symbols ($\frac{1}{3}$) on the various “parts” provided on the worksheet. The task for students is to cut out the parts of each shape at the bottom of the page and paste upon their corresponding “whole” at the top of the sheet.

5. Ask students to draw pictures to represent other unit fractions using a format of their choice:
   - e.g. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{10}$

6. Consolidate this activity by using BLM 5.3 where fractions are represented on various shapes and number lines. To complete this worksheet, students must determine how many parts in total, what fraction of the shape is shaded and what fraction of the shape is unshaded.

7. Ask students to close their eyes and visualise certain fractions, such as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{10}$ and so on. Ask them to describe the visual image that comes to their mind: Is it a circle, or a square, or a number line...or does it depend upon the fraction given?

8. Ask students to mentally calculate various part/whole calculations with unit fractions:
   - e.g., $1 - \frac{1}{3}$, $1 - \frac{1}{2}$, $1 - \frac{1}{6}$, $1 - \frac{1}{3}$, $1 - \frac{1}{4}$
5.2 EXPLORING EQUIVALENCE
(SIMPLE FRACTION ADDITION AND SUBTRACTION)

AIM: To promote simple addition and subtraction fraction mental computation through visualization of a whole comprising a number of equal parts.

OVERVIEW: In this activity, students create a set of fraction material and perform various calculations through manipulation of the fraction pieces. Through simple addition and subtraction exercises using fifths, students will encounter fraction equivalence, improper fractions and simplification.

MATERIALS: BLM 5.4 (one copy per student), scissors

TEACHING POINTS
• Encourage students to refer back to their fraction pieces if their solutions are incorrect.
• Encourage students to visualise the process of creating wholes using their fraction parts, and how many whole shapes are created when the solution is greater than one.
• Encourage students to discuss their strategies for solution to the mental calculations.

ASSESSING PROGRESS
• Students readily discuss their visual images of fraction parts and wholes when adding or subtracting.
• Students describe how the sum of parts can be greater than the whole.
• Students' mental computation becomes more accurate.

PRACTICE EXAMPLES
\[
\begin{align*}
\frac{1}{5} + \frac{2}{5} & = \frac{6}{5} = \frac{3}{4} + \frac{3}{4} \\
\frac{1}{3} + \frac{1}{3} & = \frac{5}{3} = \frac{4}{5} + \frac{2}{5} \\
\frac{1}{4} + \frac{2}{4} & = \frac{5}{2} = \frac{3}{4} + \frac{2}{4} \\
\frac{6}{7} - \frac{2}{7} & = \frac{4}{3} = \frac{5}{6} + \frac{2}{6} \\
\frac{4}{5} - \frac{1}{5} & = \frac{6}{3} = \frac{2}{3} + \frac{2}{3}
\end{align*}
\]
ACTIVITY OUTLINE:

1. Three of the circles on BLM 5.4 are clearly divided into fifths. Establish students' part/whole fraction knowledge - that each part is recorded as 1/5. Instruct students to label each fraction piece (1/5) and to cut out the three circles so that they have a total of fifteen fifths. Students may also like to write the fraction name in words (one-fifth) on the back of each fraction piece. Instruct students to cut out the whole circle and label 'one whole'.

2. Use fraction pieces to model various addition and subtraction calculations: (For each calculation, ensure students are placing the fifths in a way to make complete circles)

\[
\begin{align*}
\frac{1}{5} + \frac{2}{5} & = 1 - \frac{1}{5} \\
\frac{2}{5} + \frac{2}{5} & = 1 - \frac{3}{5} \\
\frac{2}{5} + \frac{5}{5} & = 1 - \frac{5}{5} \\
\frac{1}{5} + \frac{1}{5} & = \frac{4}{5} - \frac{2}{5} \\
\frac{4}{5} + \frac{1}{5} & = \frac{3}{5} - \frac{2}{5}
\end{align*}
\]

Use the one whole as a reference point. Put fifths on top to cover the whole.

3. Explore improper fractions and mixed numbers. Encourage students to state the answer in terms of complete 'wholes' and remaining fifths:

\[
\begin{align*}
\frac{8}{5} = & \quad \frac{4}{5} + \frac{2}{5} \\
\frac{10}{5} = & \quad \frac{3}{5} + \frac{2}{5} \\
\frac{5}{5} = & \quad \frac{2}{5} + \frac{5}{5} \\
\frac{12}{5} = & \quad \frac{3}{5} + \frac{4}{5} \\
\frac{9}{5} = & \quad \frac{4}{5} + \frac{4}{5}
\end{align*}
\]

4. Ask students to close their eyes and visualise solutions to similar fraction exercises before they record their answers:

\[
\begin{align*}
\frac{1}{5} + \frac{3}{5} & = \frac{4}{5} - \frac{2}{5} \quad \frac{7}{5} = \quad \frac{3}{5} + \frac{2}{5} \\
\frac{1}{5} + \frac{1}{5} & = 1 - \frac{4}{5} \quad \frac{5}{5} = \quad \frac{4}{5} + \frac{2}{5} \\
\frac{2}{5} + \frac{1}{5} & = \frac{3}{5} - \frac{1}{5} \quad \frac{9}{5} = \quad \frac{1}{5} + \frac{5}{5} \\
\frac{4}{5} + \frac{1}{5} & = \frac{2}{5} - \frac{1}{5} \quad \frac{10}{5} = \quad 1 + \frac{1}{5} \\
\frac{2}{5} + \frac{2}{5} & = 1 - \frac{3}{5} \quad \frac{8}{5} = \quad \frac{4}{5} + \frac{6}{5}
\end{align*}
\]

5. Explore similar operations with other fraction denominators:

\[
\begin{align*}
\frac{1}{4} + \frac{1}{4} & = 2 - \frac{1}{8} \quad \frac{1}{6} = \quad \frac{3}{4} + \frac{2}{4} \\
\frac{1}{4} + \frac{2}{4} & = \frac{9}{10} - \frac{4}{10} \quad \frac{3}{2} = \quad \frac{5}{6} + \frac{2}{6} \\
\frac{7}{8} - \frac{3}{8} & = \frac{1}{6} + \frac{1}{6} \quad \frac{5}{2} = \quad \frac{9}{10} + \frac{4}{10} \\
\frac{1}{3} + \frac{2}{3} & = 1 - \frac{3}{4} \quad \frac{5}{4} = \quad \frac{3}{2} + \frac{3}{2} \\
\frac{7}{10} + \frac{2}{10} & = \frac{6}{7} - \frac{3}{7} \quad \frac{7}{4} = \quad 2 - \frac{2}{3}
\end{align*}
\]
5.3 VISUALISING UNIT FRACTIONS WITHIN COLLECTIONS

AIM: To promote mental computation of a unit fraction within a set through visualization. This activity also links to basic division facts (e.g., 20 ÷ 4; 1/4 of 20).

OVERVIEW: In this activity, a number of counters are used to represent one whole, and students explore parts within a set (or collection of objects).

MATERIALS: Approximately 20 counters per student (unifix cubes, or other suitable material)

TEACHING POINTS:
- Continue posing problems with the material until students no longer need the material to achieve the solution.
- To assist students focus on the whole, have students place their counters on a piece of coloured paper or card with all other counters away to the side of the desk.
- Encourage students to discuss their strategies for solution to the mental calculations.

ASSESSING PROGRESS:
- Students readily discuss their visual images of a set of material that is divided into equal groups.
- Students' mental computation becomes more accurate.

PRACTICE EXAMPLES:

\[
\begin{array}{cc}
\frac{1}{2} \text{ of } 12 & \frac{1}{3} \text{ of } 9 \\
\frac{1}{4} \text{ of } 16 & \frac{1}{6} \text{ of } 24 \\
\frac{1}{4} \text{ of } 8 & \frac{1}{5} \text{ of } 20 \\
\frac{1}{3} \text{ of } 12 & \frac{1}{3} \text{ of } 9 \\
\frac{1}{5} \text{ of } 15 & \frac{1}{4} \text{ of } 20 \\
\end{array}
\]
**Activity Outline:**

1. Instruct students to put 12 counters in front of them.

2. Ask students to encase their counters with their hands, and say to the teacher, This is one whole.

3. Ask students to identify $\frac{1}{4}$ of the counters and discuss their solution strategies. Reflect on the process required - that the 12 counters had to be arranged into four groups, and each group represented one-quarter. Taking one-quarter of the whole in their hand, they could see that they had 3 counters. Therefore, one-quarter of 12 is 3.

![Illustration of 12 counters encased in a hand]

This is one whole

Four equal groups

4. Continue in this fashion with counters of various totals:

- 8 counters, show $\frac{1}{2}$
- 20 counters, show $\frac{1}{2}$
- 10 counters, show $\frac{1}{5}$
- 15 counters, show $\frac{1}{3}$
- 9 counters, show $\frac{1}{3}$

- 16 counters, show $\frac{1}{4}$
- 12 counters, show $\frac{1}{3}$
- 12 counters, show $\frac{1}{4}$
- 16 counters, show $\frac{1}{8}$
- 15 counters, show $\frac{1}{5}$

5. Ask students to close their eyes and mentally calculate various unit fractions of a group:

- $\frac{1}{5}$ of 15
- $\frac{1}{5}$ of 12
- $\frac{1}{4}$ of 8
- $\frac{1}{5}$ of 20
- $\frac{1}{4}$ of 20

- $\frac{1}{3}$ of 9
- $\frac{1}{3}$ of 12
- $\frac{1}{6}$ of 12
- $\frac{1}{10}$ of 20
- $\frac{1}{3}$ of 15

6. Ask students to explain their thinking and the mental images that come into their head.
5.4 VISUALISING NON-UNIT FRACTIONS WITHIN COLLECTIONS

AIM: To build on the visualization strategy for finding unit fractions in a set (e.g., \(\frac{1}{4}\) of 20) to finding non-unit fractions in a set through multiplication (e.g., \(\frac{3}{4}\) of 20 = \(\frac{1}{4}\) of 20 \(\times\) 3).

OVERVIEW: Linking to 5.3, in this activity students use counters to explore non-unit fractions within a set (or collection).

MATERIALS: Approximately 20 counters per student (unifix, cubes, or other suitable material)

TEACHING POINTS:
- To assist students focus on the whole, have students place their counters on a piece of coloured paper or card with all other counters away to the side of the desk.
- Ensure students are at a level of competence in finding unit fractions of the set before moving on to other fractions.
- Discuss with students the extra level of difficulty required to find fractions of a set that are not unit fractions.
- Encourage students to discuss their strategies for solution to the mental calculations.

ASSESSING PROGRESS:
- Students readily discuss their visual images of a set of material that is divided into equal groups.
- Students' mental computation becomes more accurate.

PRACTICE EXAMPLES:
- \(\frac{3}{4}\) of 12
- \(\frac{4}{5}\) of 20
- \(\frac{2}{3}\) of 15
- \(\frac{2}{3}\) of 9
- \(\frac{2}{5}\) of 20

- \(\frac{3}{4}\) of 8
- \(\frac{2}{3}\) of 6
- \(\frac{3}{5}\) of 20
- \(\frac{4}{5}\) of 25
- \(\frac{3}{4}\) of 20
**ACTIVITY OUTLINE:**

1. Follow a similar sequence as in 5.3, but draw students' attention to the extra level of difficulty required to find a fraction that is more than a unit fraction. Begin with 12 counters. Show $\frac{3}{4}$.

   Discuss the procedure required - that the 12 counters had to be arranged into four groups, and each group represented one-quarter. They were required to select three of the four groups, and that gave a total of 9.

   ![Diagram of 12 counters, 4 groups, and 3 groups highlighted]

2. Continue in this fashion with counters of various totals:
   - e.g., 12 counters, show $\frac{2}{3}$
   - 20 counters, show $\frac{3}{5}$
   - 10 counters, show $\frac{2}{5}$
   - 15 counters, show $\frac{4}{5}$
   - 20 counters, show $\frac{3}{4}$

3. Ask students to close their eyes and mentally calculate various non-unit fractions of a group:
   - $\frac{2}{5}$ of 15
   - $\frac{3}{4}$ of 16
   - $\frac{3}{4}$ of 12
   - $\frac{2}{3}$ of 20
   - $\frac{2}{3}$ of 6
   - $\frac{3}{4}$ of 9
   - $\frac{2}{5}$ of 10
   - $\frac{3}{4}$ of 20

4. Ask students to explain their thinking and the mental images that come into their head.
5.5 SIMPLE FRACTION MULTIPLICATION

AIM: To promote mental computation of fraction multiplication through visualization of movements along a number line.

OVERVIEW: In this activity, students explore simple fraction multiplication through skip counting on a number line.

MATERIALS: BLM 5.5 (one copy per student)

TEACHING POINTS:

• Encourage students to continue to display each situation on the number line until they can visualise the solution.
• Discuss the meaning of the symbolic representation as $4 \times \frac{1}{3}$ meaning 4 thirds.
• Make links to 5.1 where fraction pieces were used to explore wholes and parts, and discuss the circle as a visual image compared with the number line. Encourage students to make choices about the mental images that are most meaningful for them when performing mental calculations.

ASSESSING PROGRESS:

• Students readily discuss their visual images of fraction multiplication
• Students’ mental computation becomes more accurate.
• Students can articulate that fraction multiplication does not result in a bigger solution (as is the case with whole numbers)

PRACTICE EXAMPLES:

<table>
<thead>
<tr>
<th>$1 \times \frac{1}{2}$</th>
<th>$5 \times \frac{1}{3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3 \times \frac{1}{2}$</td>
<td>$2 \times \frac{1}{5}$</td>
</tr>
<tr>
<td>$4 \times \frac{1}{3}$</td>
<td>$5 \times \frac{1}{5}$</td>
</tr>
<tr>
<td>$6 \times \frac{1}{2}$</td>
<td>$5 \times \frac{1}{4}$</td>
</tr>
<tr>
<td>$2 \times \frac{1}{4}$</td>
<td>$5 \times \frac{1}{3}$</td>
</tr>
</tbody>
</table>
ACTIVITY OUTLINE:

1. Practice skip counting orally in thirds: $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{3}$ (1), $\frac{11}{3}$, $1\frac{2}{3}$, ...

2. Draw a number line on the board and mark in thirds, identifying the position of the numbers one and two. Demonstrate the skip counting pattern for counting in thirds by moving finger along the number line as each fraction is said.

3. Distribute BLM 5.5. Direct students to record the missing fractions on the number line as they repeat the skip counting sequence for thirds.

4. Skip count in quarters, fifths, halves, and complete the number lines on BLM 5.5. Encourage students to count aloud as they draw the sequence on the number line. Complete the blank number lines on 5.5 with fractions of your choice.

5. Using the halves number line, point-count along in halves, six times. Draw students’ attention to the position they have reached on the number line (3).

6. Write the symbolic calculation that represents the action undertaken:
   e.g., $6 \times \frac{1}{2}$
   Explain that the action was to count in halves, six times, to give a solution of 3.

7. Practice other calculations in a similar fashion, using the completed number lines to track to action:
   e.g., $5 \times \frac{1}{3}$
   $6 \times \frac{1}{4}$
   $8 \times \frac{1}{2}$
   $7 \times \frac{1}{4}$
   $4 \times \frac{1}{3}$

8. Pose similar situations, but ask students to close their eyes and visualise the number line to reach the solution.

9. Discuss visual pictures and strategies students used to arrive at the answer.

10. Practice some mental calculations of fraction multiplication.
5.6 VISUALISING FRACTION DIVISION

AIM: To promote students’ simple fraction division mental computation through visualization of fractional positions on a number line.

OVERVIEW: In this activity, students explore the number of fractional parts within wholes through reference to number lines.

MATERIALS: BLM 5.6 (one copy per student)

TEACHING POINTS:
- Encourage students to continue to display each situation on the number line until they can visualise the solution.
- Reinforce the language necessary to interpret the symbolic representation: $3 \div \frac{1}{2}$ is read as “How many halves in 3?”
- Contrast this to the language for fraction multiplication $4 \times \frac{1}{2}$ and compare the difference in solution size.
- Make links to the difference in use of the number line for fraction multiplication (see 5.5) as for fraction division.

ASSESSING PROGRESS:
- Students readily discuss their visual images of fraction division.
- Students can articulate that fraction division does not result in a smaller solution (as is the case with whole numbers).
- Students’ mental computation becomes more accurate.

PRACTICE EXAMPLES:

- $2 \div \frac{1}{2}$
- $2 \div \frac{1}{3}$
- $1 \div \frac{1}{4}$
- $3 \div \frac{1}{2}$
- $1 \div \frac{1}{3}$

- $2 \div \frac{1}{5}$
- $3 \div \frac{1}{4}$
- $3 \div \frac{1}{5}$
- $1 \div \frac{1}{10}$
- $2 \div \frac{1}{4}$
ACTIVITY OUTLINE:

1. Ask students to close their eyes and visualize a number line for counting in halves. Ask them to mentally locate the position of the following numbers: \(\frac{1}{2}, 2\frac{1}{2}, 1, 6, 3\frac{1}{2}\).

2. Distribute BLM 5.6. Direct students' attention to the halves number line, and ask them to visualize how many halves are in one, and how it might be drawn on that number line.

3. Draw a possible representation on the board to show how many halves are in one:

   ![Number Line Representation]

4. Invite students to share other possible representations on the number line. Discuss the merits of each representation. Have students show that there are 2 halves in one on the number line on BLM 5.6.

5. Ask students to add to their picture to show that there are four halves in two.

6. Ask students to fill in missing numbers from other number lines on 5.6.

7. Ask students to use the number lines to determine the following:
   - how many thirds in 2
   - how many quarters (fourths) in 2
   - how many thirds in 4
   - how many fifths in 2

8. Ask students to consider how many halves in 3. Write the symbolic recording that represents the action undertaken:

   \[3 \div \frac{1}{2}\]

   Encourage students to read this as ‘How many halves in 3?’

9. Write similar equations on the board, and ask students to close their eyes and visualize the solution. Discuss whether the number line assisted them to attain the solution.

10. Practice some mental calculations of fraction division.
5.7 VISUALISING DECIMALS TENTHS

AIM: To promote visualisation of a number line when performing mental computation of addition and subtraction in tenths to one whole.

OVERVIEW: In this activity, students use a number line to explore decimal tenths.

MATERIALS: BLM 5.7 (one copy per student)

TEACHING POINTS:

- Discuss the features of the number line for addition and subtraction. Link this action to similar counting activities using a number line.
- Link this activity to students’ strategies for basic 10s facts and discuss similarities (bonds) and differences (total of one, not 10).

ASSESSING PROGRESS

- Students readily discuss their visual images of decimal addition and subtraction to one.
- Students’ mental computation becomes more accurate.

PRACTICE EXAMPLES:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 + 0.5</td>
<td>1 - 0.7</td>
</tr>
<tr>
<td>0.2 + 0.8</td>
<td>1 - 0.2</td>
</tr>
<tr>
<td>0.6 + 0.4</td>
<td>1 - 0.3</td>
</tr>
<tr>
<td>0.3 + 0.2</td>
<td>1 - 0.9</td>
</tr>
<tr>
<td>0.4 + 0.7</td>
<td>1 - 0.5</td>
</tr>
</tbody>
</table>
**ACTIVITY OUTLINE:**

1. Practice skip counting in decimal tenths 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1, 1.1..., using the language point one, point two..., rather than its appropriate fraction name, to differentiate between fraction number lines.

2. Listen for any students saying point ten after point nine, and discuss this possible misconception. Make the link to fraction tenths to assist in developing meaning.

3. Distribute BLM 5.7. Direct students to fill in missing decimals on the number line as they repeat the skip counting sequence on the number line showing tenths.

![Number Line](image)

4. Pose some addition and subtraction computations of tenths to one. e.g., 0.3 + 0.7, 0.3 + 0.5, 1 - 0.6, 1 - 0.4, .6 + 0.3

5. Pose similar situations, but ask students to close their eyes and visualise the number line to reach the solution.

6. Discuss visual pictures and strategies students use to arrive at the solution.

7. Practice some mental calculations of decimal addition and subtraction to one.
AIM: To promote mental computation of decimal multiplication through visualization of movements along a number line.

OVERVIEW: In this activity, students explore simple decimal multiplication through skip counting on a number line.

MATERIALS: BLM 5.8 (one copy per student), calculators

TEACHING POINTS:

- Encourage students to continue to use the number line until they can visualise the solution.
- Discuss the language of the symbolic recording: \(3 \times 0.5\) is counting in 0.5s.

ASSESSING PROGRESS:

- Students readily discuss their visual images of decimal multiplication.
- Students can articulate that decimal multiplication does not result in a bigger solution (as in whole number multiplication).
- Students can link this activity with fraction multiplication.
- Students' mental computation becomes more accurate.

PRACTICE EXAMPLES:

\[
\begin{align*}
2 \times 0.5 & \quad 7 \times 0.2 \\
3 \times 0.5 & \quad 3 \times 0.6 \\
6 \times 0.5 & \quad 4 \times 0.7 \\
5 \times 0.2 & \quad 5 \times 0.7 \\
8 \times 0.2 & \quad 5 \times 0.4 \\
\end{align*}
\]
ACTIVITY OUTLINE:

1. Practice skip counting in decimal fifths: 0.5, 1, 1.5, 2, 2.5...

2. Draw a number line on the board and mark in decimal fifths. Demonstrate the skip pattern by marking in the ‘skips’ along the number line as each decimal is said.

3. Distribute BLM 5.8. Direct students to fill in the missing numbers on the number line

```
0     0.5     1     1.5    2      2.5    3
```

4. Have students observe the skip counting pattern on the calculator:

   Clear calculator, enter 0.5, then press ‘+’ button twice, and then the ‘=’ button. Check that the ‘1’ is displayed. Ask students to press the ‘=’ button again to see if the calculator is performing a counting action. Ask students to continue to press the ‘=’ button and observe the counting sequence.

   Discuss the movement of the decimal point on the screen.

5. Using the 0.5 number line, ask students to count in 0.5s, six times, tracking the sequence with their fingers. Draw students’ attention to the position they have reached on the number line (3).

6. Write the symbolic calculation that represents the action undertaken:

   e.g.,   6 x 0.5

   Explain the action was to count in 0.5, six times, to give a solution of 3. Draw students’ attention to strategies they used to find fraction multiplication.

7. Practise other calculations in a similar fashion, using the number line to track the action:

   e.g.   5 x 0.5
          4 x 0.2
          8 x 0.5
          5 x 0.4
          2 x 0.6

8. Pose similar situations, but ask students to close their eyes and visualise the number line to reach the solution.

9. Discuss visual pictures and strategies students used to arrive at the answer.

10. Practice some mental calculations of decimal multiplication.
5.9 **VISUALISING DECIMAL DIVISION**

**AIM:** To promote students’ simple decimal division mental computation through visualization of decimal positions on a number line.

**OVERVIEW:** In this activity, students explore the number of decimal parts within a whole through reference to a number line.

**MATERIALS:** BLM 5.9 (one copy per student), calculators

**TEACHING POINTS:**
- Encourage students to continue to use the number line until they can visualise the solution.
- Reinforce the language necessary to interpret the symbolic representation: \(3 \div 0.5\) is read as *How many 0.5s in 3?*
- Encourage students to verbalise the strategy of thinking of decimal sections within 1 as a reference point for thinking of numbers greater than one.

**ASSESSING PROGRESS:**
- Students readily discuss their visual images of decimal division.
- Students can articulate that decimal division does not result in a smaller solution (as in whole number division).
- Students can link this activity with fraction division.
- Students’ mental computation becomes more accurate.

**PRACTICE EXAMPLES:**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>Extension</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(2 \div 0.5)</td>
<td>(1.5 \div 0.5)</td>
<td>(1.5 \div 0.5)</td>
<td></td>
</tr>
<tr>
<td>(3 \div 0.5)</td>
<td>(2.5 \div 0.5)</td>
<td>(2.5 \div 0.5)</td>
<td></td>
</tr>
<tr>
<td>(1 \div 0.2)</td>
<td>(1.2 \div 0.2)</td>
<td>(2 \div 0.4)</td>
<td></td>
</tr>
<tr>
<td>(2 \div 0.2)</td>
<td>(1.6 \div 0.2)</td>
<td>(1.8 \div 0.3)</td>
<td></td>
</tr>
<tr>
<td>(4 \div 0.2)</td>
<td>(0.8 \div 0.2)</td>
<td>(1.2 \div 0.3)</td>
<td></td>
</tr>
</tbody>
</table>
ACTIVITY OUTLINE:

1. Ask students to close their eyes and visualise a number line marked in 0.5 sections. Ask them to mentally locate the following points on the number line:

   0.5, 2.5, 1, 0, 6, 3.5.

2. Distribute BLM 5.9. Direct students’ attention to the number line marked in 0.5 sections, and ask them to state how many 0.5 sections are in one. Discuss how this might be drawn on a number line.

3. Draw a possible representation on the board:

   ![Number Line Diagram]

4. Have students record this (or their own) representation on the number line on BLM 5.9.

5. Ask students to add to their picture to show that there are four 0.5s in two.

6. Ask students to use the number lines to determine solutions to the following:

   - how many 0.5s in 2
   - how many 0.5s in 5
   - how many 0.2s in 1
   - how many 0.2s in 2
   - how many 0.2s in 6

7. Ask students to consider how many 0.5s in 2 and write the symbolic representation on the board:

   \[2 \div 0.2\]

   Encourage students to read this as How many 0.5s in 2...

8. Write similar situations on the board, but ask students to close their eyes and visualise the solution. Discuss whether the number line assisted them to attain the solution and what thinking they engaged in.

   - \[3 \div 0.5\]
     means how many 0.5s in 3...
     Locate 2 on the number line
     There are two 0.5s in 1,

9. Practise some mental calculations of decimal division.
BLM 5.1
Circles
Cut out the shapes at the bottom of the page.
Write the fraction name on each shape.
Paste the thirds on the shapes below to make whole shapes.
BLM 5.3
**Fraction parts and wholes**

Look at each shape or number line.

a) Write how many parts in each shape.
b) Write the fraction name for the number of parts that are shaded.
c) Write the fraction name for the number of parts that are unshaded.

<table>
<thead>
<tr>
<th>Shape 1</th>
<th>Shape 2</th>
<th>Shape 3</th>
<th>Shape 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Shape 1" /></td>
<td><img src="image2.png" alt="Shape 2" /></td>
<td><img src="image3.png" alt="Shape 3" /></td>
<td><img src="image4.png" alt="Shape 4" /></td>
</tr>
</tbody>
</table>

1. Number of parts in total = _____
2. Fraction of parts shaded = _____
3. Fraction of parts unshaded = _____

1. Number of parts in total = _____
2. Fraction of parts shaded = _____
3. Fraction of parts unshaded = _____

1. Number of parts in total = _____
2. Fraction of parts shaded = _____
3. Fraction of parts unshaded = _____

1. Number of parts in total = _____
2. Fraction of parts shaded = _____
3. Fraction of parts unshaded = _____

1. Number of parts in total = _____
2. Fraction of parts shaded = _____
3. Fraction of parts unshaded = _____

1. Number of parts in total = _____
2. Fraction of parts shaded = _____
3. Fraction of parts unshaded = _____
Fraction Fifths
BLM 5.5

Skip counting number lines

Thirsts

Quarters

Fifths

Halves
BLM 5.6

Pictures for Fraction Division

Halves

Thirds

Quarters

Thirds

Fifths
Tenths Number Lines

0        1                    2
0        1                    2
0        1                    2
0        1                    2
0        1                    2
0        1                    2
0        1                    2
0        1                    2
Pictures for Decimal Fraction Division
Fractions & Decimals
Shelley Dole
The University of Queensland

What are fractions?
Fractions are numbers, and belong to the set of rational numbers. Decimals are also rational numbers. Whole numbers, counting numbers and integers (positive and negative numbers) are also a subset of the numbers we call rational.
Fractions can be located on a number line, and are generally thought of as parts of a whole.

Fraction comparisons
A lot of fraction comparison can occur through evoking mental images. For example, consider the following fraction pairs, and think about what came into your head when you were trying to compare them.

Which is bigger:
   a) 1/3 or 1/5?
   b) ¾ or ⅝
   c) 2/9 or 4/9
   d) 2/3 or 2/5
   e) 3/7 or 5/9

Did some mental pictures come into your head? What were they?
When building students’ conceptual knowledge of fractions, we should always assist them to be able to build strong mental pictures so that they do not have to resort to computation or calculation to determine the size of fractions.

Meanings of fractions
   part of a whole
   part of a set
   division
   ratio
   operator (that which multiplies by numerator and divides by denominator)

Part of a whole fraction meaning
The steps for developing fraction understanding are as follows:
(i) identify the whole (“This is one whole”)
(ii) partition into equal parts (divide into 3 equal parts)
(iii) name the parts (each part is 1/3)
(iv) determine the number of parts (I need 2 parts)
(v) name the fraction (this is 2/3)

“Reversing” - a teaching strategy
When we teach, we must be careful that our teaching is not “one way”. That is, we should not always ask students to perform the same types of operations. For example, we often
ask students to partition a “whole” into an equal number of parts. We also should go the other way (reverse), by showing students a model of a particular number of parts, and asking them to construct the whole. Try it! Get a piece of paper, and show me three-quarters. Then take another piece of paper and tell yourself that this piece of paper represents two-thirds. Now create the original whole. Reversing should not only be integral to teaching fractions, but all other maths topics as well.

**Teaching model**

As always, we must pay attention to the three modes of representation when we teach any concept - concrete, oral, symbolic. This is particularly important as we include a new fraction name for “fourths” - quarters. We also need to ensure that students know what the two numbers in the symbolic fraction representation stand for. For example, in the fraction \( \frac{3}{4} \), the bottom number (denominator) tells us how many parts the whole has been divided into, and the top number (numerator) tells us how many parts we have taken.

**Fraction models**

(i) Region  
e.g., common 2D shapes - circle, square, rectangle, triangle, etc  
need to show students a variety of these shapes so that they don’t see that “piece of a pie” all the time.

(ii) Set  
e.g., a collection of objects - counters, blocks, etc

(iii) Length  
e.g., string, paper strips, leading to number line

(iv) Area  
These models are similar to region models, but the whole is divided into equal parts that have the same area, but not necessarily the same shape.

**Real world fraction models**

Take care with using real world fraction models (cakes, pies, pizzas, licorice) as they are rarely divided into equal pieces in the real world.

**Sequencing of activities**

(i) Partitioning - the whole is cut into parts (sharing)  
(ii) Words - number of equal parts; halves, thirds, fourths/quarters  
(iii) Counting - one half, two halves - one whole...; one third, two thirds, three thirds - one whole, one and one third...  
(iv) Drawing a model - rectangles are easiest to partition  
(v) Extending a model - to show equivalent fractions. E.g., draw a diagram to show two thirds; then show four sixths on the same diagram.

**Developing part of a whole conceptual understanding**
Provide students with many opportunities to divide different models into parts (and also parts back to wholes). Consider the positive and negatives of various models, for example, the rectangle is easy to partition, but students can lose sight of the whole as they are working with the different “bits” of paper; circles are easiest to reconstruct and focus on the whole, but are most difficult to partition into any fractions other than halves and quarters.

**Fraction circles**

Make two sets of fraction material using the circles printed on white paper, and the 5 sheets of coloured paper.

(i) *Sixths*

Cut out one white circle and 2 other circles of different colours.

Make one circle into two halves, and write the name of each piece on each piece in words on one side (one third), and in symbols on the other \( \left( \frac{1}{3} \right) \). Do this for the other coloured circle, this time making the pieces into sixths.

Use the fraction pieces to explore various part/whole fraction actions.

E.g., How many thirds make one whole?
- How many more pieces to make one whole if I have two thirds?
- How many wholes if I have 4 thirds?
- What is 2 thirds and 2 thirds?
- How many sixths in one third?
- How many sixths in two thirds?
- How many sixths in one whole?

Make up other simple equivalences, additions and subtractions that can be carried out using the fraction set. Describe the value for children in manipulating such sets of material (and making them for themselves). Discuss the mental pictures that such materials can develop. Discuss the value of the circle as a model.

(ii) *Eighths*

Cut out one white circle and 3 other circles of different colours.

Make one circle into two halves, and write the name of each piece on each piece in words on one side (one half), and in symbols on the other \( \left( \frac{1}{2} \right) \). Do this for the other two coloured circles, this time making the pieces into quarters and eighths respectively.

Explore this set of fraction material in the same way as you did for the sixths. Would it be necessary to have children create fraction sets for all denominators up to 10? Why/why not?

**Fraction strips**

Make a set of fraction strips for tenths, fifths, halves and one whole (see over).

Use different coloured paper for each fraction size.

Write the names on each piece.

Do similar activities as for the fraction circles.

Discuss the value of this model.
Discuss the value of this exploration in terms of its link to decimals.

<table>
<thead>
<tr>
<th>one whole</th>
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<tr>
<td>one half</td>
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<tr>
<td>one fifth</td>
</tr>
<tr>
<td>one tenth</td>
</tr>
</tbody>
</table>

**Addition and subtraction of fractions**

Encourage mental images; work with fraction pieces so that students see that there are, for example, 4 quarters in a whole; 5 quarters mean that there is one whole and one quarter left over; that one half is the same as two quarters; that one fifth is smaller than one third.

For addition and subtraction, students should be able to “see” the additions and subtractions in their head, without having to resort to calculation. Encourage mental computation, such as

\[ 1 - \frac{1}{3} \]
\[ \frac{3}{5} + \frac{1}{5} \]
\[ \frac{3}{4} + \frac{3}{4} \]
\[ \frac{3}{7} + \frac{4}{7} = 1 \text{ whole} \]

**Part of a set**

Students need to see the connection between fractions of one whole, and fractions of a set. They need to see that the process is essentially the same, and that the entire set is the “whole”. This cannot be over-emphasised enough as many students have difficulty with determining such calculations as one third of 6; two-fifths of 20, and the like.

The teaching sequence for part of a set is the same as for part of a whole, and must begin with students focusing of the whole. Placing the objects on a piece of paper, or in a circle of string can assist this beginning point.

Using counters, practise finding the *part* given a *whole* (e.g., get 12 counters and find one third). Conversely, practise finding the whole, given a part (e.g., put out 4 counters, and note that this represents one quarter. Add counters so that you have the whole). Think about the cognitive levels required for each activity. Sequence the level of difficulty. Make up further examples according to difficulty level.

Examples:

- 12 cubes is one whole; show \(\frac{1}{3}\).
- 18 cubes is one whole; show \(\frac{1}{6}\).
- 20 cubes is one whole; show \(\frac{1}{4}\).
- 15 cubes is one whole; show \(\frac{2}{3}\).

- 4 cubes is \(\frac{1}{4}\); show one whole
3 cubes is \( \frac{1}{2} \); show one whole
2 cubes is \( \frac{1}{5} \); show one and three fifths

20 cubes is one and one quarter; show one whole
15 cubes is one and two thirds; show one whole
21 cubes is one and three quarters; show one whole
10 cubes is two and a half; show one whole

If students are operating at an computational level, they may find the following activities more of a challenge. Those students who were operating at a conceptual level will have more chance of finding the solution. Working with the counters to act out the solution process will assist in reaching the answer.

*If you have a set of counters that totals 16, and you are told that the 16 is the same as one and a third, can you tell me how many counters would make up one quarter? (Hint: the answer is not 4).*

Here are some more of this type:
- 20 cubes is one and two-thirds; show one quarter
- 15 cubes is one and a half; show two fifths
- 20 cubes is one and a quarter; show two and a half
- 24 cubes is one and one fifth; show three quarters
- 21 counters is one and three quarters; show two thirds
- 24 counters is one and one third; show one half

Here is a real world application of the knowledge required for the above situation:

*A farmer bought a small farm. To do this, he borrowed \( \frac{3}{4} \) of the price, interest-free, from his father in law. Two years later, he sold the farm for \( \frac{2}{3} \) more than he paid for it. The selling price was \$200 000. How much did he have to repay his father-in-law.*

**Fractions as division**

Consider the problem 3 divided by 5. Put it into a real context, and see what happens.

E.g., share 3 pizzas between 5 people. Take 3 circles and discuss how the three pizzas could be cut up and shared. If you do this by cutting each circle into fifths, and sharing the fifths between the 5 people, you will see that each person gets three fifths. So what has this shown? That 3 divided by 5 is the same as three fifths. This action shows that the operation is division and that it is partition division (sharing, dealing).

Students need explicit teaching to develop the concept of fractions as division. They need to see that a fraction written in symbolic form means the top number divided by the bottom number. This is easier to see when we consider improper fractions. E.g., \( \frac{4}{3} \) means 4 divided by 3, and hence the “rule” we learnt that to change an improper fraction into a mixed number, look at the bottom number and say “how many threes go into four”. Keep thinking of the pizza division - for this example, we have 4 pizzas and three people. Each person gets 1 whole pizza and one third of the extra pizza.
Fraction multiplication
How can fraction multiplication be modelled meaningfully? One way to start thinking about this, is to think about whole number multiplication. How do we encourage children to think of 3x4? Do we say 3 groups of 4? Try this for fractions, but select your examples carefully. E.g., 3 x ½ could be read as :3 groups of one half. What pictures might be drawn to represent this? Try other fraction combinations.

Think about using a number line. When we show 3x4 on a number line, we jump the number line in fours, three times (assuming we read the number sentence as three groups of 4). Could we use this representation for 3 x ½? Try it. Draw some pictures; draw some number lines. Encourage students to discuss various situations and solutions with others in your group.

6 x ½ makes more sense when you say “6 groups of ½”. Draw a diagram to show that the solution is 3.

Fraction division
How can fraction division be modeled meaningfully? How do we read 6÷½? How would we read such a situation if we were using whole numbers? For 6÷2 could we say: How many twos in six? Could we say something similar for 6÷½? How many halves in 6? Could we use a number line and see how many halves are in one, and then keep going until we found out how many halves in six?

6 ÷ ½ makes more sense (and hopefully a mental picture) when we say “How many halves in 6?”. A number line also may help. There are two halves in one, so there are 12 halves in 6.

Decimals and fraction knowledge
Developing decimal understanding builds on prior knowledge from (i) numeration knowledge and (ii) fraction knowledge.

Linking fractions knowledge to decimal understanding
Prior experience with fractions provides the basis for conceptualising decimals as fractions. The sequence is as follows:
1. Exploration of halves, fifths and tenths as a family of fractions.
2. Symbolic representation of tenths linked to visual image.
3. Symbolic representation of hundredths and tenths to visual image and number line.
A set of fractions made from paper enables the relationship between halves, fifths and tenths to be explored. When such fractions are created using a linear model, the foundation is laid for visualising decimals on a number line. Encourage manipulation of the fraction pieces until students can determine answers to the following types of questions mentally:
- How many tenths in one whole?
- What is three-tenths and three-fifths?
- How many tenths is the same as one-half?
- What is seven-tenths and nine-tenths?
- What’s another name for fifteen-tenths?

Shading pictures of tenths provides a link between the fraction picture, the symbolic fraction form and the symbolic decimal form.

\[ \frac{3}{10} = 0.3 \]

\[ 1\frac{5}{10} = 1.5 \]

The relationship between tenths and hundreds is seen through creating pictures of various fractions and decimals on 10x10 grids. From images created the relative magnitude of tenths in relation to hundredths is seen. Location of decimals on the number line follows from the pictorial representation.

Numeration knowledge
Decimal numbers differ from whole numbers in the addition of a new series of place-value positions to the right of the ones. These new decimal positions are all fractions. Hence the notion of a fraction underpins decimal numeration.

- imagine a number line, and locate the numbers 1 and 2
- now locate 0
- now locate -1 and -2
- now locate 23
- now locate 101
- now locate -99

Where are these numbers?
What can you say about numbers? (infinite)

**Conceptualising very small numbers**
- locate the half-way point between 1 and 2. What is its decimal name? (1.5)
- focus on the point whose decimal name is 1.6
- how far apart are 1.5 and 1.6? (one-tenth)
- what about the point half-way between 1.55 and 1.56?
- draw and redraw the number line to show this point

These exercises help to establish a feel for the magnitude of decimal numbers, the infinite numbers that can be located on the number line, the fluid nature of the number line, the insignificance in the decimal places beyond thousandths.

**Development of decimal number knowledge**
- interference caused by whole number knowledge:
  - zero added to rightmost column does not change total value
  - values decrease as move away from decimal point
  - column names end in "ths", not "s"
  - naming numbers start with tenths, not ones
  - naming sequence (tenths, hundredths...) moves left to right, not right to left)
  - reading sequence is tenths, hundredths, thousands

- Some students develop "rules" for decimals that are incorrect
- the number with the most decimal places is the larger one:
  - e.g., 3.214 is larger than 3.8 because 214 is greater than 8
- The number with the fewer decimal places is the larger one:
  - e.g., 1.2 is greater than 1.35 because 1.2 has fewer decimal places
PERCENT MENTALS TEST

Instructions
- Read each question twice only
- Allow about 5-8 seconds between each question
- If students do not know, ask them to put a dash on the answer sheet, and encourage them to listen to the next questions

<table>
<thead>
<tr>
<th>SET A</th>
<th>SET B</th>
<th>SET C</th>
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</thead>
<tbody>
<tr>
<td>1. 50% of 600</td>
<td>1. 10% of 50</td>
<td>1. 10% of 80</td>
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<tr>
<td>2. 50% of 17</td>
<td>2. 10% of 42</td>
<td>2. 30% of 80</td>
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<tr>
<td>3. 50% of 1 000 000</td>
<td>3. 10% of $6.50</td>
<td>3. 10% of 60</td>
</tr>
<tr>
<td>4. 25% of 800</td>
<td>4. 10% of $7 000 000</td>
<td>4. 90% of 60</td>
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<td>5. 25% of 40</td>
<td>5. 5% of 36</td>
<td>5. 70% of 700</td>
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<td>6. 25% of 50</td>
<td>6. 5% of $48</td>
<td>6. 40% of 900</td>
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<td>7. 25% of 300</td>
<td>7. 5% of 1400</td>
<td>7. 60% of 400</td>
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<tr>
<td>8. 33 1/3% of 60</td>
<td>8. 1% of 700</td>
<td>8. 30% of 90</td>
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<td>9. 33 1/3% of 150</td>
<td>9. 1% of 0.3</td>
<td>9. 20% of 200</td>
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<tr>
<td>10. 12 1/2% of 80</td>
<td>10. 100% of 200</td>
<td>10. 20% of 100</td>
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<thead>
<tr>
<th>SET D</th>
<th>SET E</th>
<th>SET F</th>
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<tbody>
<tr>
<td>Calculate new price after 10% discount</td>
<td>Increase...</td>
<td>Express as a fraction:</td>
</tr>
<tr>
<td>1. $10</td>
<td>1. 50 by 100</td>
<td>1. 25%</td>
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<tr>
<td>2. $8</td>
<td>2. 50 by 200%</td>
<td>2. 33 1/3%</td>
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<tr>
<td>3. $40</td>
<td>3. 6 by 33 1/3%</td>
<td>3. 150%</td>
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<td>4. $400</td>
<td>4. 8 by 25%</td>
<td>4. 12 1/2%</td>
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<td>5. $350</td>
<td>5. 10 by 50%</td>
<td>5. 20%</td>
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<tr>
<td>6. $16</td>
<td>6. 70 by 300%</td>
<td>6. 66%</td>
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<tr>
<td>7. $2.50</td>
<td>7. 20 by 50%</td>
<td>7. 35%</td>
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<tr>
<td>8. $4.80</td>
<td>8. 7 by 50%</td>
<td>8. 3%</td>
</tr>
<tr>
<td>9. $200</td>
<td>9. 8 by 150%</td>
<td>9. 129%</td>
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<tr>
<td>10. $5</td>
<td>10. 300 by 33 1/3%</td>
<td>10. 10%</td>
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</tbody>
</table>
PERCENT MENTALS TEST

Name: ____________________________________
Grade: _______  Date:______________

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Evidence of percent usage and applications abound in our society ensuring the importance of percent as a topic of study within the mathematics curriculum. Yet, despite its usage, percent is commonly misapplied. A simplistic view of percent is often the reason for misapplication of percent. Percent is a difficult topic to teach and to learn, but is often narrowly taught to link only to fraction and decimals. A rich conceptual understanding of percent, and confidence to use and apply percent, relies on knowledge of not only fractions and decimals, but also ratio understanding and proportional reasoning skills.

**Meanings of Percent**
Percent has many meanings and is used for different purposes in different contexts. The multifaceted nature of percent is seen in the following meanings of percent:

- Percent is a fraction: - 35% is 35/100 in the part/whole sense
- Percent is a decimal: - 35% is 0.35
- Percent is a number: - 35% can be counted and located on a number line
- Percent is a ratio: - there are 35% more girls in Class A as Class B
- Percent is a proportion: - 70 out of 200 is equivalent to 35 out of 100 which is 35%

Percent is a difficult concept because of the many meanings and the many meanings are masked by a concise symbol %.

**Building Conceptual Understanding of Percent**
The basic concept of percent is its link to a base of 100. Linking percent to fractions and decimals is a natural extension of using 10x10 grids to represent decimal (and fraction) numbers (as outlined in the section on decimals). As students draw various pictures of fractions with denominators of 100, labelling the diagram as a percent shows the equivalence of fractions, decimals and percent. Drawing particular percents in a highly visual way can also show the link of some common percents to fractions (e.g., 50% is ½, 25% is ¼, 75% is ¾, 10% is 1/10) and promote percent benchmarks.
The difficulty with using 10x10 grids for percent representation occurs when percents are greater than 1. To draw a picture of 115%, the dilemma is whether to use 2 grids when 1 is not enough. Yet this would possibly invoke the misconception of two wholes being required for percents greater than 1. A better way is to use number lines simultaneously with 10x10 grids. For percents greater than 1, the number line can be extended beyond the 100 percent mark. Representing percents on a number line is a means of promoting understanding of percent as a number.

### The Language of Percent

Percent is frequently used in everyday language and conversation, but quite often, its usage does not adhere to the mathematical rules of percent. This may be a source of confusion to students as the everyday usage of the term violates the appropriate mathematical sense of the term. It is not uncommon for young children playing sport to be told by the coach that they are expected to put in “110% effort” for the game. If 100% is used to designate the whole, can the whole be more than 100%? Engaging students in discussion of the language of percent in colloquial terms and mathematical terms is a valuable way of determining students’ intuitive notions of percent and the extent to which these notions may help or hinder further conceptual development of percent in a mathematical sense.

### Percent Part and Complement

To promote a rich understanding of percent, students need to think of percent as describing a situation in terms of a base of 100. They also need to see the situation as one part in relation to 100 and the other remaining part (the complement) in relation to 100. For example, if a label on a wool jumper states that the jumper is 80% wool then it is known that 20% of the jumper is made of fibres that are not wool; if 55% of the people voted for no homework, then 45% of the people voted for homework.

To assist students visualize percent situations in terms of the part and the complements, representing various situations on 10x10 grids is useful. By shading the various parts of the whole on the grid, a visual image is presented of the situation and the part and complement are seen as comprising the whole. Shading 10x10 grids is a simpler than using a circle graph for representing information in percentages as the 100 parts in the whole are clearly visible.

Encourage mental computation of simple percent situations and practice of calculations to 100 by asking students to close their eyes and visualise various percent part and complement situations. Prior experience of depicting percent situations on 10x10
grids provides a strong visual image for assisting in determining solutions to such things as the following:

- If I drank 25% of the milk, what percent was left?
- Twenty-eight percent of the smarties were green, what percent were not green?
- The jacket was discounted 25%; what percent of the original price did I have to pay?
- Rump steak is 82% beef; what percent is fat?
- Soap contains 0.5% perfume, 37% pure soap, and the rest is fat. What percent is fat?
- The human body is about 78% water. What percent is made up of other stuff?

**Increases of more than 100%**

Common misapplications of percent occur when percent increases of greater than 100% are given. The difficulty in conceptualising and checking calculations of increases of more than 100% lies in the fact that the situation can be considered from both an additive and a multiplicative perspective and this can cause confusion. Consider the example of a newspaper report describing how a statue purchased for $1 and then being valued at $200 was a 200% price increase. The confusion is that the original whole amount ($1) has not been considered as 100%, but has possibly been considered as a value of 1. As the original amount of $1 is equivalent to 100%, an increase of 100% would mean doubling the original amount to give a new whole of $2. A 200% increase is actually 3 times the original amount, which in this case would be $3. By considering a 200% increase in this way the magnitude of the actual percentage increase to reach $200 becomes apparent.

The actual percentage increase of the statue from $1 to $200 is actually 19,900%. Considering 100% and 200% increases as in the example above, the additive and multiplicative interpretations of percent increases can be seen. If the statue was purchased for $1 and it was resold for $2, the percent increase from an additive interpretation is 100%. That is, 100% has been added to the original whole. In a multiplicative sense, the new price is 200% of its original price. The subtle differences between interpreting increase situations in a multiplicative or additive sense helps to explain how simple errors can be made.

The multiplicative and additive interpretations of percentage increases of 100% and other multiples of 100 can be assisted through explorations with concrete materials. By taking a small group of counters, this group of counters can be increased by 100% by adding the same amount of counters as the original group. The size of this group is then compared to the original amount. An increase of 100% means the new size of the group is twice that of the original amount. When the group is increased 200%, the increase is three times the original. Through such explorations, the teacher’s role is to encourage students to generalise what happens as a collection is increased by an amount of more than 100% or multiples of 100. Students’ attention should be drawn to the fact that, for example, a 200% increase is not the same as multiplying by 2. Such investigations lay the foundation for visualising mental calculations of amounts greater than 100%. Ask students to close their eyes and visualise increasing groups by various percentages (e.g., 4 counters, increase 100%; 50 counters, increase 100%; 800 counters, increase 300%). Always encourage students to discuss their solution strategies.
**Percent increases and decreases**

From exploring percent increases of more than 100% the language foundation and multiplicative structure is emphasised in order to use the language of percent to appropriately described situations. Consider the example of a shop owner increasing prices before a sale by 25% and then giving a 25% discount. Does the shop owner win, lose, or break even in this situation? Actually, by employing this strategy, the shop owner loses.

Modelling this situation with counters exemplifies why this is so. Take a set of 8 counters. Increase this group of counters by 25% (add 2 counters). The new total amount of counters is 10. Therefore, the new whole totals 10. How many counters must be removed from this group to take it back to its original amount? (2). What fraction of the new total is this? (one-fifth). What percent is this? (20%).

From this demonstration, we see that if we increase something by 25%, we actually need to reduce it by 20% to get back to the original amount. Relating this back to the shop owner, a 25% discount after increasing the original price by 25% means that the shop owner is selling the product for less than the price before the sale. The shop owner would not have a hope of breaking even or making a profit.

Through modelling with counters, the following relationships can be discovered:

- An increase of 50% means a reduction of $33\frac{1}{3}\%$ to get back to the original.
- An increase of 25% means a reduction of 20% to get back to the original.
- An increase of $33\frac{1}{3}\%$ means a reduction of 25% to get back to the original.
- An increase of 100% means a reduction of 50% to get back to the original.

**Mental Computation of Percent Situations**

Mental computation and estimation of percent problems should have prominence in the middle years of schooling. Learning a myriad of rules and procedures does little to assist students to interpret situations in which percent is used. The development of common percent benchmarks (e.g., 50 is one half; 25% is one quarter; 10% is one tenth) lays the foundation for simple mental percent computation of finding a percentage of an amount. Percent/fraction equivalence is useful for supporting mental computation of finding a percentage of an amount. For example, to find 25% of 80, it is simple to think of 25% as one-quarter and then to divide 80 by 4. Similarly, thinking of 10% as one-tenth assists in mentally dividing the amount by 10 in order to find 10%.

Finding a percentage of an amount is the simplest of all percent calculations and students need to be provided with opportunities for mentally calculating simple percentages of this type. Instruction should focus on developing students’ skills in finding...
a percent of a number through the use of percent benchmarks (25%, 50%, 10%) and then using such skills to make closer estimates. Drawing pictures of percent situations on dual-scale number lines also assists students relate percent as proportional situations (see diagram below).

600mL carton of milk; 20% cream. How many mL of the milk is cream?

Concluding comments
Percent is a multi-faceted topic that is more than conversions and calculations. Decimal and common fraction knowledge promotes percent knowledge, but also ratio understanding and proportional reasoning skills are required for successful operation in the domain of percent. For percent instruction, students need to be provided with experiences that promote their understanding of the big ideas of percent and connections to other related topics. Exploring and analysing the language of percent is also important. Often confusion with percent is a result of rushing to symbolic representation and manipulation of numbers before a rich conceptual framework has been sufficiently developed. Confidence and skill in using and applying percents is a valuable life skill. Current teaching practices for percent instruction need to be reconsidered to ensure its relevance for students in the 21st Century.